YAPI STATİĞİ 2

DERS NOTLARI(2-3)

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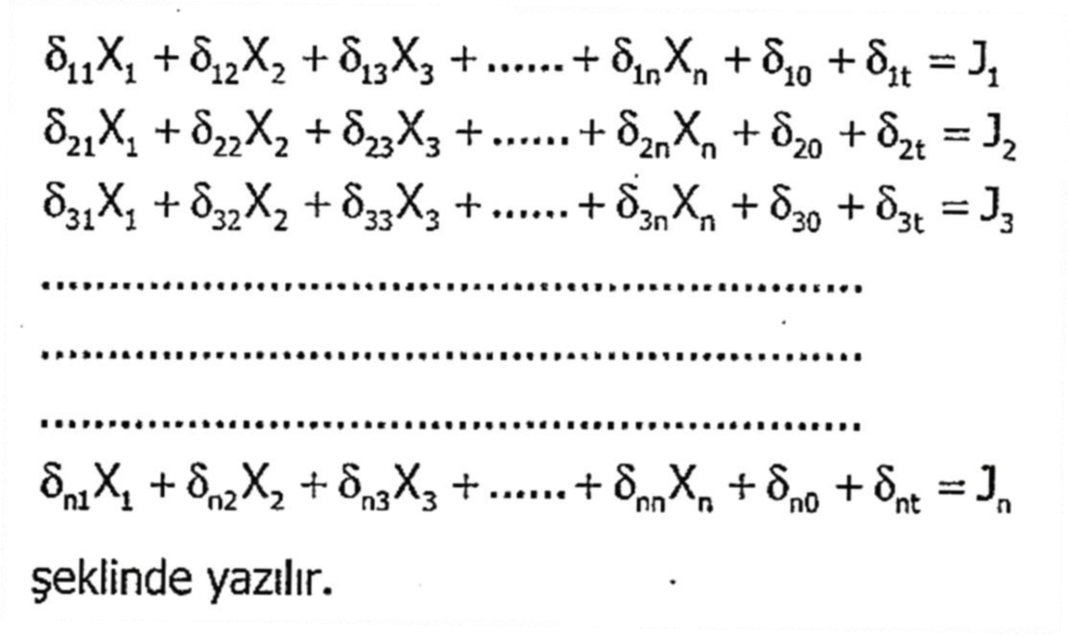
Güncelleme Tarihi: 25 Mart 2024

GENEL SÜREKLİLİK DENKLEMLERİ

- n. dereceden bir hiperstatik sisteme dış yükler, mesnet çökmeleri ve sıcaklık etkisi birlikte etki ediyorsa; Açık Süreklilik Denklemleri



$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \ldots\ldots + \delta_{1n}X_n + \delta_{10} + \delta_{1t} = J_1$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \ldots\ldots + \delta_{2n}X_n + \delta_{20} + \delta_{2t} = J_2$$

$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \ldots\ldots + \delta_{3n}X_n + \delta_{30} + \delta_{3t} = J_3$$

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$$\delta_{n1}X_1 + \delta_{n2}X_2 + \delta_{n3}X_3 + \ldots\ldots + \delta_{nn}X_n + \delta_{n0} + \delta_{nt} = J_n$$

şeklinde yazılır.

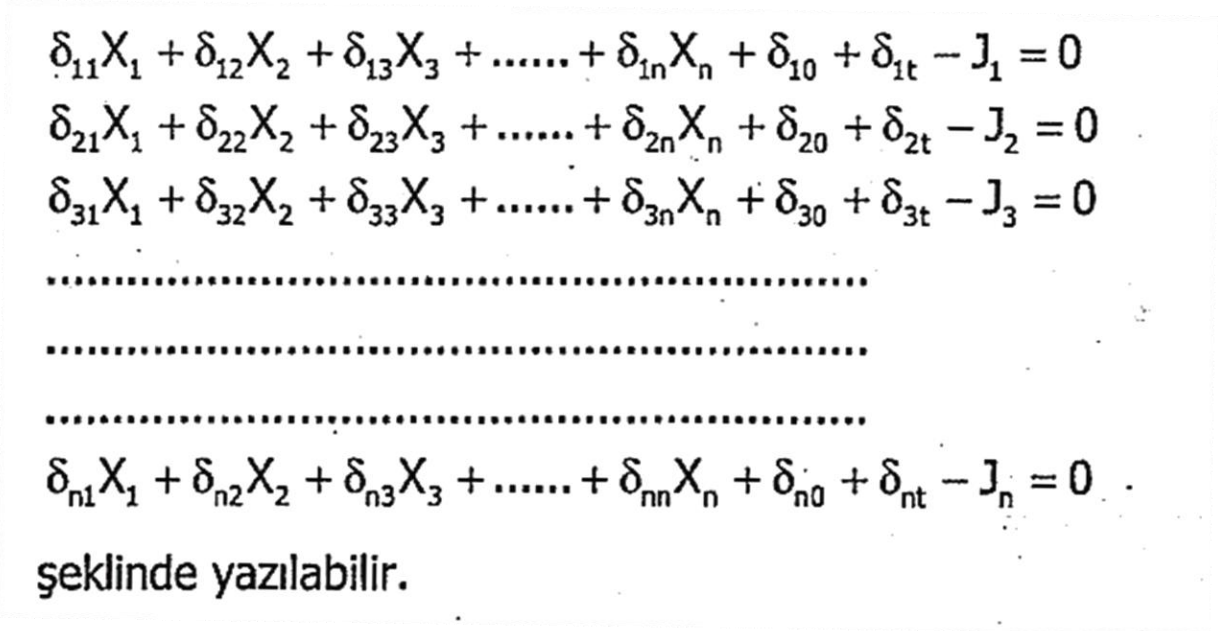
- n. dereceden bir hiperstatik sisteme dış yükler, mesnet çökmeleri ve sıcaklık etkisi birlikte etki ediyorsa; Kapalı Süreklilik Denklemleri

$$\int \mathbf{M}_i \frac{\mathbf{M}}{\mathbf{EI}}\mathbf{ds} + \int \mathbf{N}_i \frac{\mathbf{N}}{\mathbf{EF}}\mathbf{ds} + \int \mathbf{T}_i \frac{\mathbf{T}}{\mathbf{GF'}}\mathbf{ds} + \int \mathbf{M}_i \frac{\boldsymbol{\varepsilon}\boldsymbol{\Delta}\mathbf{t}}{\boldsymbol{d}}\mathbf{ds} + \int \mathbf{N}_i \boldsymbol{\varepsilon}\mathbf{t}_s\mathbf{ds} = \mathbf{J}_i \ \textbf{i nolu kapalı süreklilik denklemi}$$


olarak elde edilir.

d=kesit yüksekliği

Açık Süreklilik Denklemlerinin Matris Formunda Yazılışı:


$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \ldots\ldots + \delta_{1n}X_n + \delta_{10} + \delta_{1t} - J_1 = 0$$
$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \ldots\ldots + \delta_{2n}X_n + \delta_{20} + \delta_{2t} - J_2 = 0$$
$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \ldots\ldots + \delta_{3n}X_n + \delta_{30} + \delta_{3t} - J_3 = 0$$

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$$\delta_{n1}X_1 + \delta_{n2}X_2 + \delta_{n3}X_3 + \ldots\ldots + \delta_{nn}X_n + \delta_{n0} + \delta_{nt} - J_n = 0$$

şeklinde yazılabilir.

$$\textbf{Katsayılar matrisi}[\Delta] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \dots & \dots & \dots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \delta_{23} & \dots & \dots & \dots & \delta_{2n} \\ \delta_{31} & \delta_{32} & \delta_{33} & \dots & \dots & \dots & \delta_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \delta_{n1} & \delta_{n2} & \delta_{n3} & \dots & \dots & \dots & \delta_{nn} \end{bmatrix}_{(n \times n)} \quad \text{(kare matris)}$$

$$\text{Bilinmeyenler matrisi : } [X] = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \dots \\ \dots \\ \dots \\ X_n \end{bmatrix}_{(n \times 1)} \quad \text{(kolon matris)}$$

$$\text{Sabitler matrisi : } [\delta_0] = \begin{bmatrix} \delta_{10} + \delta_{1t} - J_1 \\ \delta_{20} + \delta_{2t} - J_2 \\ \delta_{30} + \delta_{3t} - J_3 \\ \dots \\ \dots \\ \dots \\ \delta_{n0} + \delta_{nt} - J_n \end{bmatrix}_{(n \times 1)} \quad \text{(kolon matris)}$$

olduğuna göre, Açık Süreklilik Denklemlerinin matris formunda ifadesi

$$[\Delta][\boldsymbol{X}] + [\boldsymbol{\delta_0}] = \mathbf{0}$$

şeklinde yazılır.

$$[\mathbf{X}] = -[\Delta]^{-1}[\boldsymbol{\delta_0}] \qquad [\boldsymbol{\beta}] = -[\Delta]^{-1} \rightarrow [\mathbf{X}] = [\boldsymbol{\beta}]\,[\boldsymbol{\delta_0}]$$

$$[\beta] = \begin{bmatrix} \beta_{11} & \beta_{12} \dots & \beta_{1n} \\ \beta_{21} & \beta_{22} \dots & \beta_{2n} \\ \dots \dots & \dots \dots & \dots \dots \\ \beta_{n1} & \beta_{n2} \dots & \beta_{nn} \end{bmatrix}$$

$$\boldsymbol{\beta_{ij}} = (-\mathbf{1})(-\mathbf{1})^{i+j}\frac{\Delta_{ij}}{\Delta} \qquad \boldsymbol{\beta_{ij}} = \boldsymbol{\beta_{ji}}$$

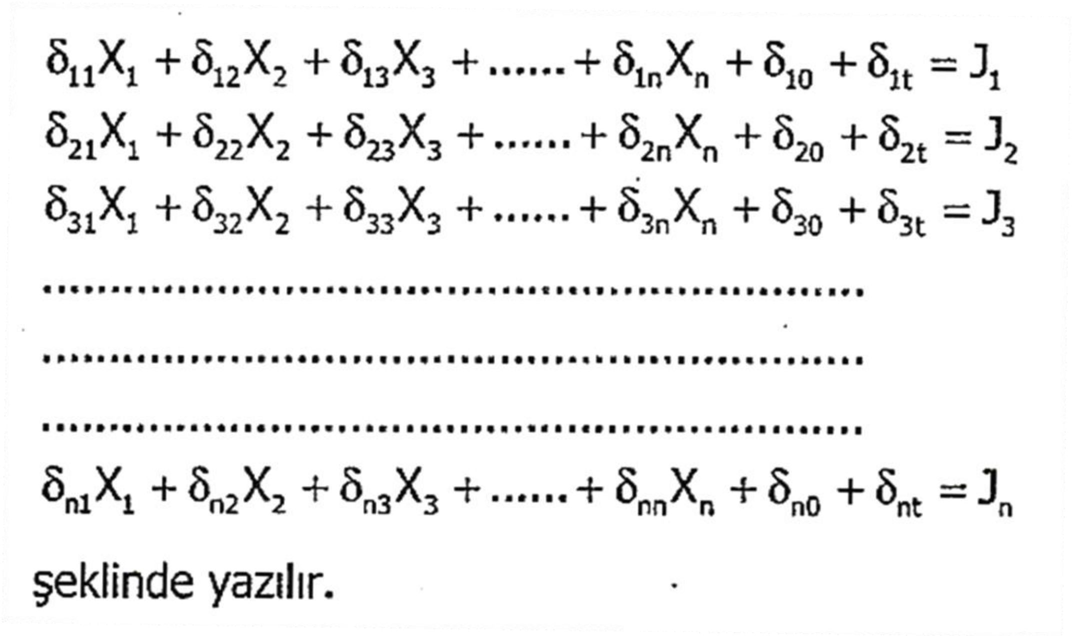
n. Dereceden hiperstatik sistemde süperpozisyon denklemleri

$$\mathbf{M = M_0 + M_1X_1 + M_2X_2 + \cdots + M_nX_n = M_0 + \sum_{i=1}^{n} X_i}$$

$$\mathbf{N= N_0 + N_1X_1 + N_2X_2 + \cdots + N_nX_n = N_0 + \sum_{i=1}^{n} N_iX_i}$$

$$\mathbf{T= T_0 + T_1X_1 + T_2X_2 + \cdots + T_nX_n = T_0 + \sum_{i=1}^{n} T_iX_i}$$

Süperpozisyon denklemleri



$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \ldots\ldots + \delta_{1n}X_n + \delta_{10} + \delta_{1t} = J_1$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \ldots\ldots + \delta_{2n}X_n + \delta_{20} + \delta_{2t} = J_2$$

$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \ldots\ldots + \delta_{3n}X_n + \delta_{30} + \delta_{3t} = J_3$$

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$$\delta_{n1}X_1 + \delta_{n2}X_2 + \delta_{n3}X_3 + \ldots\ldots + \delta_{nn}X_n + \delta_{n0} + \delta_{nt} = J_n$$

şeklinde yazılır.

Açık süreklilik denklemleri

$$\int M_i \frac{M}{EI} ds + \int N_i \frac{N}{EF} ds + \int T_i \frac{T}{GF'} ds + \int M_i \frac{\varepsilon \Delta t}{d} ds + \int N_i \varepsilon t_s ds = J_i \text{ i nolu kapalı süreklilik denklemi}$$

$$\delta_{ij} = \int M_i \frac{M_j}{EI} ds + \int N_i \frac{N_j}{EF} ds + \int T_i \frac{T_j}{GF'} ds \quad i = 1 \ldots \ldots . n \quad j = 1 \ldots \ldots \ldots . n$$

$$\delta_{i0} = \int M_i \frac{M_0}{EI} ds + \int N_i \frac{N_0}{EF} ds + \int T_i \frac{T_j}{GF'} ds \quad i = 1 \ldots \ldots . n \quad j = 1 \ldots \ldots \ldots . n$$

$$\delta_{it} = \int M_i \frac{\varepsilon \Delta t}{h} ds + \int N_i \varepsilon ds \quad i = 1 \ldots \ldots \ldots . n$$

Daima δ_{ij} δ_{i0} δ_{1t} ve J_i nin EI_c katları alınarak X_i hiperstatik bilinmeyenler çözülür.

$$EI_c\delta_{ij} = \int M_i M_j \left[\frac{I_c}{I}\right] ds \ldots . . \qquad EI_c\delta_{i0} = \int M_i M_0 \left[\frac{I_c}{I}\right] ds \ldots . . \qquad EI_c J_i = EI_c * J_i$$

$$EI_c\delta_{it} = EI_c \int M_i \frac{\varepsilon \Delta t}{d} ds + EI_c \int N_i \varepsilon t_s ds \quad i = 1 \ldots \ldots \ldots . n$$

n. Dereceden hiperstatik kafes sistemde süperpozisyon denklemi

$$S = S_0 + S_1X_1 + S_2X_2 + \cdots . + S_nX_n = S_0 + \sum_{i=1}^{n} S_iX_i$$

İ inci kapalı süreklilik denklemi

$$\sum S_i \frac{S}{EF} L + \sum S_i \varepsilon t_s L = J_i$$

$$\delta_{ij} = \sum S_i S_j \frac{L}{EF}$$

$$EF_c\delta_{ij} = \sum S_i S_j [\frac{F_c}{F}] L$$

$$\delta_{i0} = \sum S_i S_0 \frac{L}{EF}$$

$$EF_c\delta_{i0} = \sum S_i S_0 [\frac{F_c}{F}] L$$

$$EF_c\delta_{it} = \sum [S_i \varepsilon t_s L] EF_c$$

$$\delta_{it} = \sum S_i \varepsilon t_s L$$

$$EF_cJ_i = J_i * EF_c$$

S_i : X_i=1 birim yüklemeden dolayı meydana gelen çubuk kuvveti

S_j : X_j=1 birim yüklemeden dolayı meydana gelen çubuk kuvveti

J_i : X_i=1 birim yüklemesinden dolayı mesnet tepkilerinin mesnet çökmelerinde yapmış olduğu iş

F_c : En büyük kesit alanı

β matrisinin özellikleri

1. $\beta_{ij} = \beta_{ji}$ simetrik
2. Köşegen üzerinde olan terimler negatiftir.
3. β matrisinin herhangi bir satırı ile Δ matrisinin aynı satırının çarpımı -1 dir.

$$\boldsymbol{\beta_{11}\delta_{11} + \beta_{12}\delta_{12} + \beta_{13}\delta_{13} + \cdots + \beta_{1n}\delta_{1n} = -1}$$

4. β matrisinin herhangi bir satırı ile Δ matrisinin farklı bir satırının çarpımı (0) dır.

$$\boldsymbol{\beta_{11}\delta_{21} + \beta_{12}\delta_{22} + \beta_{13}\delta_{23} + \cdots + \beta_{1n}\delta_{2n} = 0}$$

$$\mathbf{X_1} = \boldsymbol{\beta_{11}}\mathbf{EI_c}\boldsymbol{\delta_{10}} + \boldsymbol{\beta_{12}}\mathbf{EI_c}\boldsymbol{\delta_{20}} + \cdots + \boldsymbol{\beta_{1n}}\mathbf{EI_c}\boldsymbol{\delta_{n0}}$$
$$\mathbf{X_2} = \boldsymbol{\beta_{21}}\mathbf{EI_c}\boldsymbol{\delta_{10}} + \boldsymbol{\beta_{22}}\mathbf{EI_c}\boldsymbol{\delta_{20}} + \cdots + \boldsymbol{\beta_{2n}}\mathbf{EI_c}\boldsymbol{\delta_{n0}}$$

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$$\mathbf{X_n} = \boldsymbol{\beta_{n1}}\mathbf{EI_c}\boldsymbol{\delta_{10}} + \boldsymbol{\beta_{n2}}\mathbf{EI_c}\boldsymbol{\delta_{20}} + \cdots + \boldsymbol{\beta_{nn}}\mathbf{EI_c}\boldsymbol{\delta_{n0}}$$

$$\boldsymbol{\beta_{ij}} = (-\mathbf{1})(-\mathbf{1})^{\mathbf{i+j}}\frac{\Delta_{\mathbf{ij}}}{\Delta} \quad \boldsymbol{\beta_{ij}} = \boldsymbol{\beta_{ji}}$$

1. Sistem üzerinde sadece dış yükler varsa

$$\mathbf{X_i^d} = \boldsymbol{\beta}_{\mathbf{i1}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{10}} + \boldsymbol{\beta}_{\mathbf{i2}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{20}} + \cdots + \boldsymbol{\beta}_{\mathbf{in}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{n0}} \quad \mathbf{i = 1 \ldots\ldots n}$$

$$\boldsymbol{\beta_{ij}} = (-\mathbf{1})(-\mathbf{1})^{i+j}\frac{\Delta_{ij}}{\Delta} \quad \boldsymbol{\beta_{ij}} = \boldsymbol{\beta_{ji}}$$

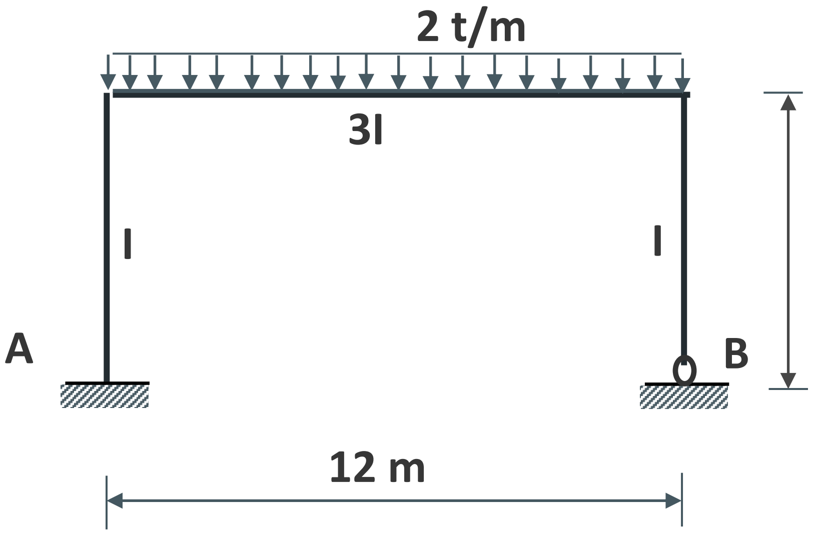
2. Sistemde sadece sıcaklık değişimi varsa

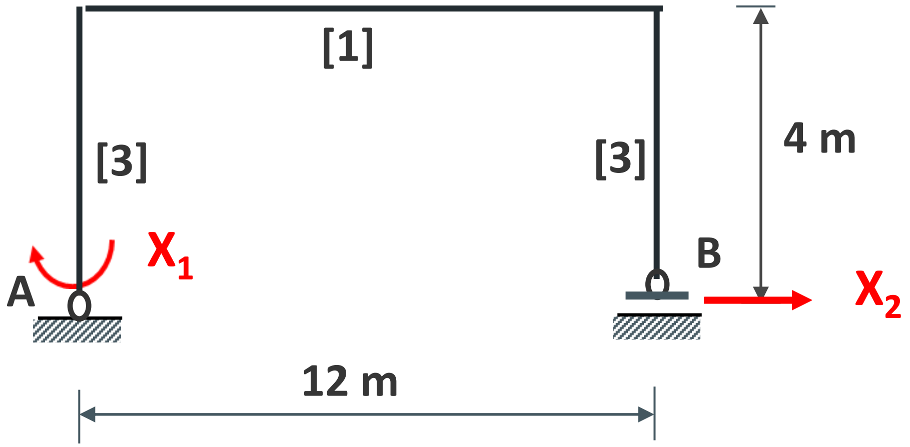
$$\mathbf{X_i^t} = \boldsymbol{\beta}_{\mathbf{i1}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{1t}} + \boldsymbol{\beta}_{\mathbf{i2}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{2t}} + \cdots + \boldsymbol{\beta}_{\mathbf{in}}\mathbf{EI_c}\boldsymbol{\delta}_{\mathbf{nt}} \quad \mathbf{i = 1 \ldots \ldots n}$$

3. Sistemde sadece mesnet çökmeleri varsa

$$\mathbf{X_i^m} = \overline{\boldsymbol{\beta}}_{\mathbf{i1}}\mathbf{EI_cJ_1} + \overline{\boldsymbol{\beta}}_{\mathbf{i2}}\mathbf{EI_cJ_2} + \cdots + \overline{\boldsymbol{\beta}}_{\mathbf{in}}\mathbf{EI_cJ_n} \qquad \mathbf{i = 1 \ldots \ldots n}$$

$$\overline{\boldsymbol{\beta}}_{\mathbf{ij}} = (-\mathbf{1})^{\mathbf{i+j}}\frac{\boldsymbol{\Delta}_{\mathbf{ij}}}{\Delta} \quad \overline{\boldsymbol{\beta}}_{\mathbf{ij}} = \overline{\boldsymbol{\beta}}_{\mathbf{ji}}$$





β_{ij} leri tayin ederek hiperstatik bilinmeyenleri bulunuz.

$$EI_c\underbrace{\begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}}_{\boldsymbol{\delta}}\begin{Bmatrix}X_1\\ X_2\end{Bmatrix}+\underbrace{EI_c\begin{Bmatrix}\delta_{10}\\ \delta_{20}\end{Bmatrix}}_{\boldsymbol{\delta_0}}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$16X_1+48X_2+144=0$$

$$48X_1+320X_2+1152=0$$

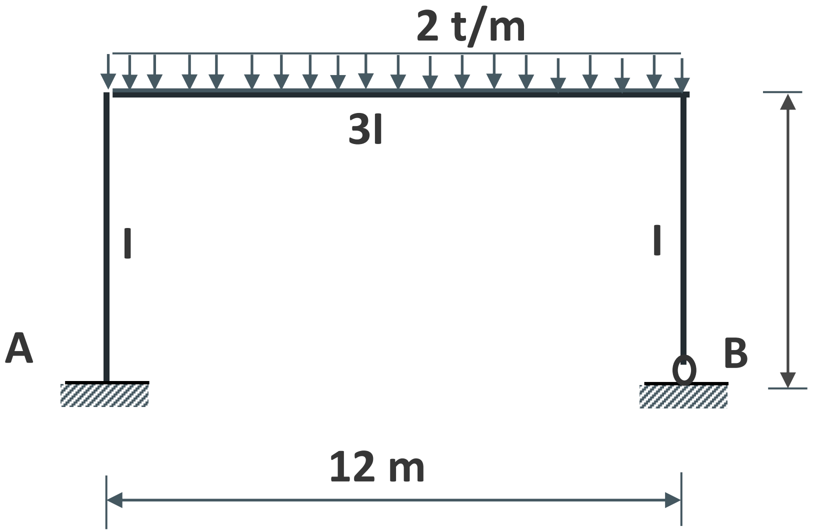
$$\Delta=\begin{vmatrix}\mathbf{16} & \mathbf{48}\\ \mathbf{48} & \mathbf{320}\end{vmatrix}=\mathbf{2816}$$

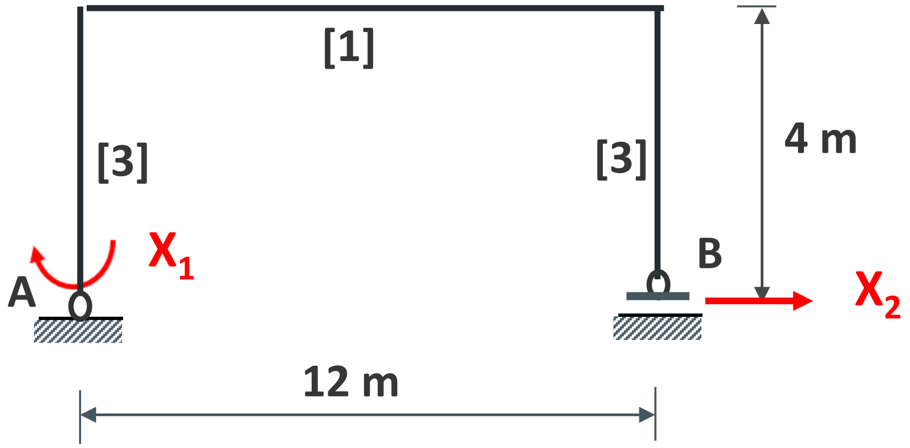
$$\boldsymbol{\beta_{ij}=(-1)(-1)^{i+j}\frac{\Delta_{ij}}{\Delta}} \qquad \boldsymbol{\beta_{ij}=\beta_{ji}}$$

$$\beta_{11}=(-1)(-1)^{1+1}\frac{320}{2816}=-11.36*10^{-2}$$

$$\beta_{12}=(-1)(-1)^{1+2}\frac{48}{2816}=-1.704*10^{-2}=\beta_{21}$$

$$\beta_{22}=(-1)(-1)^{2+2}\frac{16}{2816}=-0.568*10^{-2}$$





$$\boldsymbol{\beta_{ij} = (-1)(-1)^{i+j}\frac{\Delta_{ij}}{\Delta} \quad \beta_{ij} = \beta_{ji}}$$

$$\beta_{11} = (-1)(-1)^{1+1}\frac{320}{2816} = -11.36 * 10^{-2}$$

$$\beta_{12} = (-1)(-1)^{1+2}\frac{48}{2816} = -1.704 * 10^{-2} = \beta_{21}$$

$$\beta_{22} = (-1)(-1)^{2+2}\frac{16}{2816} = -0.568 * 10^{-2}$$

$$\beta = \begin{bmatrix} -11.36 * 10^{-2} & 1.704 * 10^{-2} \\ 1.704 * 10^{-2} & -0.568 * 10^{-2} \end{bmatrix}$$

$$\boldsymbol{X_1 = \beta_{11}EI_c\delta_{10} + \beta_{12}EI_c\delta_{20}}$$
$$\boldsymbol{X_2 = \beta_{21}EI_c\delta_{10} + \beta_{22}EI_c\delta_{20}}$$

$$X_1 = (-11.36 * 10^{-2} * 144 + 1.704 * 10^{-2} * 1152 = 3.24 \text{ tm}$$

$$X_2 = (1.704 * 10^{-2} * 144 + (-0.568) * 10^{-2} * 1152 = -4.08 \text{ tm}$$

$$\boldsymbol{\beta_{11}\delta_{11} + \beta_{12}\delta_{12} = -1 \text{ olmalı}}$$

$$-11.36 * 10^{-2} * 16 + 1.704 * 10^{-2} * 48 = -1 \sqrt{}$$

$$\boldsymbol{\beta_{11}\delta_{21} + \beta_{12}\delta_{22} = 0 \text{ olmalı}}$$

$$-11.36 * 10^{-2} * 48 + 1.704 * 10^{-2} * 320 = 0 \sqrt{}$$

$$EI_c \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + EI_c \begin{Bmatrix} \delta_{10} \\ \delta_{20} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\mathrm{EI_c}\delta = \begin{bmatrix} 16 & 48 \\ 48 & 320 \end{bmatrix}$$

$$\beta \rightarrow \frac{\beta}{\mathrm{EI_c}}$$

$$16X_1 + 48X_2 + 144 = 0$$

$$48X_1 + 320X_2 + 1152 = 0$$

$$\beta = \frac{1}{\mathrm{EI_c}} \begin{bmatrix} -11.36 * 10^{-2} & 1.704 * 10^{-2} \\ 1.704 * 10^{-2} & -0.568 * 10^{-2} \end{bmatrix}$$

$$\mathbf{X_1 = \beta_{11}EI_c\delta_{10} + \beta_{12}EI_c\delta_{20}}$$

$$\mathbf{X_2 = \beta_{21}EI_c\delta_{10} + \beta_{22}EI_c\delta_{20}}$$

$$X_1 = (-11.36 * 10^{-2} * 144 + 1.704 * 10^{-2} * 1152 = 3.24 \text{ tm}$$

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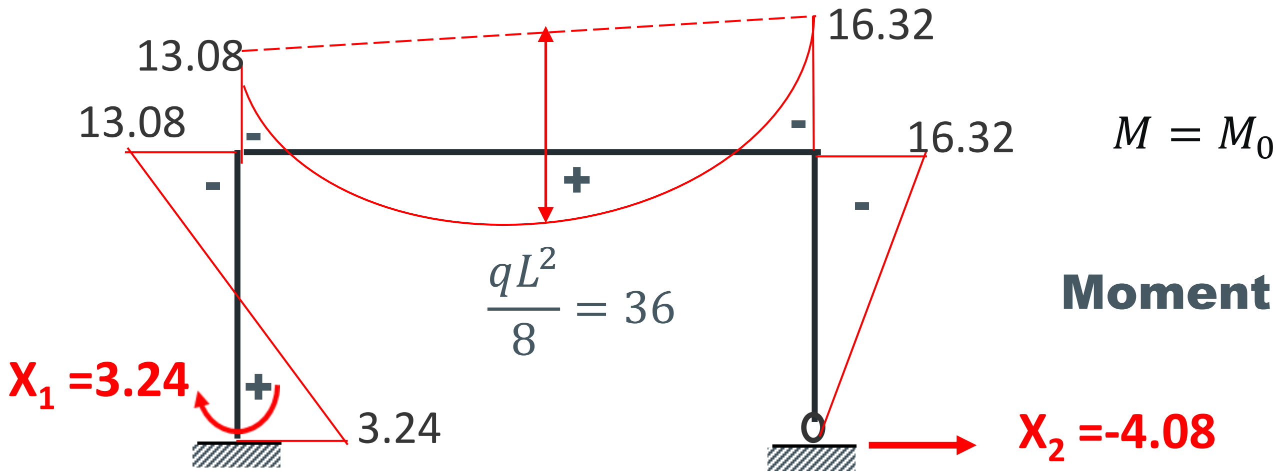
$$\boldsymbol{\beta_{11}\delta_{11} + \beta_{12}\delta_{12} = -1} \textbf{ olmalı} \qquad -11.36 * 10^{-2} * 16 + 1.704 * 10^{-2} * 48 = -1 \surd$$

$$\boldsymbol{\beta_{11}\delta_{21} + \beta_{12}\delta_{22} = 0} \textbf{ olmalı} \qquad -11.36 * 10^{-2} * 48 + 1.704 * 10^{-2} * 320 = 0 \surd$$



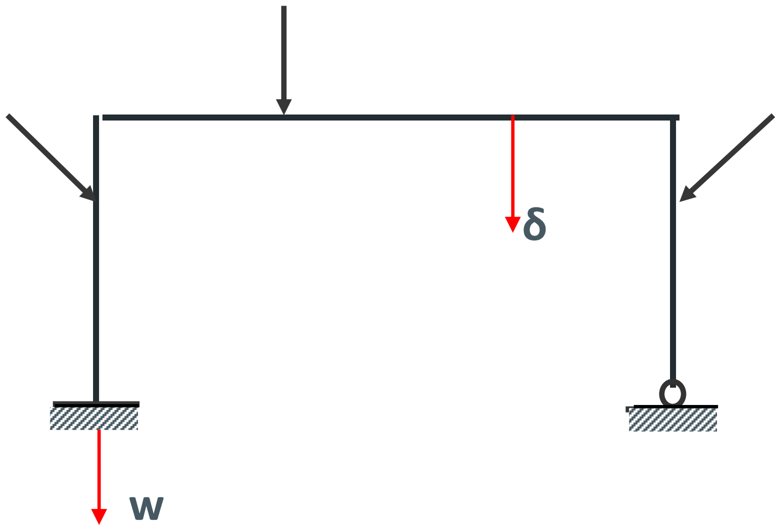
$$M = M_0 + M_1X_1 + M_2X_2$$

Moment diyagramı

HİPERSTATİK SİSTEMLERDE DEPLASMAN HESABI

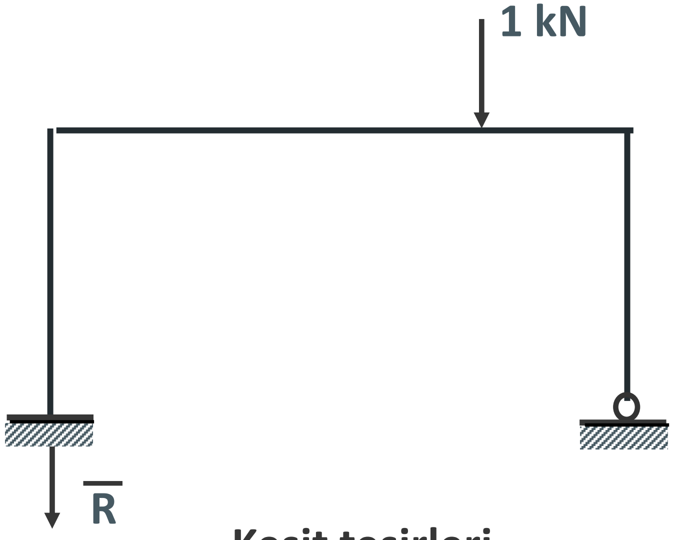
1. Virtüel iş prensibi ile
2. Mohr metodu ile

1. VİRTÜEL İŞ PRENSİBİ İLE DEPLASMAN HESABI



Deplasman sebepleri : dış yük, t_s Δt, w

Deformasyon durumu



Kesit tesirleri

$\overline{M}$ $\overline{N}$ $\overline{T}$

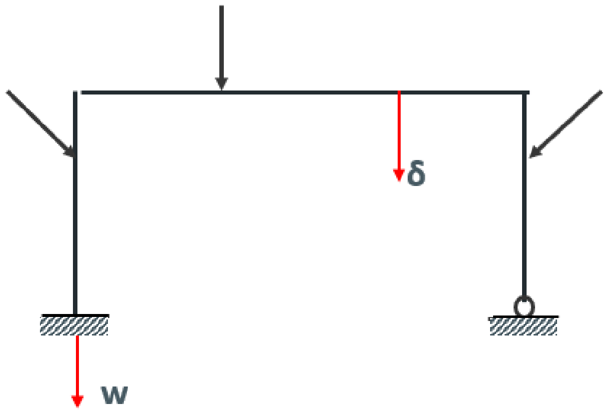
Yükleme durumu

w: mesnet çökmeleri

Deformasyonlar $\Delta\varphi = \frac{M}{EI}ds + \frac{\varepsilon\Delta t}{d}ds$

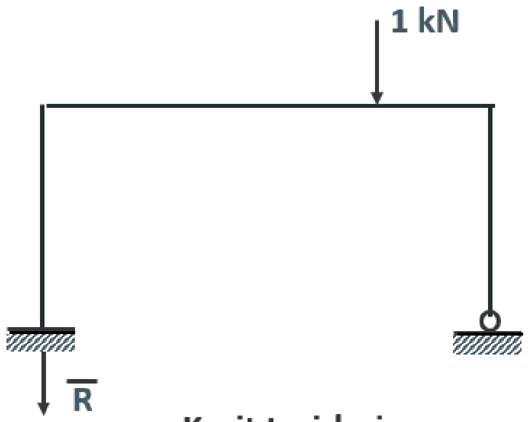
$$\Delta ds = \frac{N}{EF}ds + \varepsilon t_s ds$$

$$\Delta v = \frac{T}{GF'}ds$$



Deplasman sebepleri : dış yük, t_s Δt, w

Deformasyon durumu



Kesit tesirleri

$\overline{M}$ $\overline{N}$ $\overline{T}$

Yükleme durumu

Dış kuvvetlerin yaptığı iş = iç kuvvetlerin yaptığı iş

$$1.\delta + \overline{R}w = \int \overline{M}\Delta\varphi + \int \overline{N}\Delta ds + \int \overline{T}\Delta v$$

$$1.\delta + \overline{R}w = \int \overline{M}(\frac{M}{EI}ds + \frac{\varepsilon\Delta t}{h}ds) + \int \overline{N}\left(\frac{N}{EF}ds + \varepsilon t_s ds\right) + \int \overline{T}\frac{T}{GF'}ds$$

$$\boxed{1.\delta + \overline{R}w = \int \overline{M}\frac{M}{EI}ds + \int \overline{N}\frac{N}{EF}ds + \int \overline{T}\frac{T}{GF'}ds + \int \overline{M}\frac{\varepsilon\Delta t}{d}ds + \int \overline{N}\varepsilon t_s ds \qquad \textbf{(I)}}$$

(M N T) ve ($\overline{M}$ $\overline{N}$ $\overline{T}$) her ikisi de aynı hiperstatik sisteme ait

KISALTMA TEOREMİ

Eğer (M, N, T) ve $(\overline{M}, \overline{N}, \overline{T})$ gruplarının her ikisi de hiperstatik sisteme aitse bu gruplardan herhangi birini bu hiperstatik sisteme ait herhangi bir İzostatik esas sistemden almak mümkündür.

$$
\textbf{(II)}\;\begin{cases}
\overline{M} = \overline{M}_0 + M_1\overline{X_1} + M_2\overline{X_2} + \cdots + M_n\overline{X_n} = \displaystyle\sum_{i=1}^{n} M_i\overline{X_i} + \overline{M}_0 \\
\overline{N} = \overline{N}_0 + N_1\overline{X_1} + N_2\overline{X_2} + \cdots + N_n\overline{X_n} = \displaystyle\sum_{i=1}^{n} N_i\overline{X_i} + \overline{N}_0 \\
\overline{T} = \overline{T}_0 + \overline{T}_1 X_1 + T_2\overline{X_2} + \cdots + T_n\overline{X_n} = \displaystyle\sum_{i=1}^{n} T_i\overline{X_i} + \overline{T}_0 \\
\overline{R} = \overline{R}_0 + R_1\overline{X_1} + R_2\overline{X_2} + \cdots + R_n\overline{X_n} = \displaystyle\sum_{i=1}^{n} R_i\overline{X_i} + \overline{R}_0
\end{cases}
$$

$$1.\delta + \overline{R}w = \int \overline{M}\frac{M}{EI}ds + \int \overline{N}\frac{N}{EF}ds + \int \overline{T}\frac{T}{GF'}ds + \int \overline{M}\frac{\varepsilon\Delta t}{d}ds + \int \overline{N}\varepsilon t_s ds \qquad \textbf{(I)}$$

$$\left.\begin{aligned}
\overline{M} &= \overline{M_0} + M_1\overline{X_1} + M_2\overline{X_2} + \cdots + M_n\overline{X_n} = \sum_{i=1}^{n} M_i\overline{X_i} + \overline{M_0} \\
\overline{N} &= \overline{N_0} + N_1\overline{X_1} + N_2\overline{X_2} + \cdots + N_n\overline{X_n} = \sum_{i=1}^{n} N_i\overline{X_i} + \overline{N_0} \\
\overline{T} &= \overline{T_0} + T_1\overline{X_1} + T_2\overline{X_2} + \cdots + T_n\overline{X_n} = \sum_{i=1}^{n} T_i\overline{X_i} + \overline{T_0} \\
\overline{R} &= \overline{R_0} + R_1\overline{X_1} + R_2\overline{X_2} + \cdots + R_n\overline{X_n} = \sum_{i=1}^{n} R_i\overline{X_i} + \overline{R_0}
\end{aligned}\right\} \qquad \textbf{(II)}$$

(II) denklemini (I) de yerine koyarsak

$$1\,.\,\delta + \overline{R_0}w + \left(\sum R_i\overline{X_i}\right)w = \int\left(\overline{M_0} + \sum M_i\overline{X_i}\right)\frac{M}{EI}ds + \int\left(\overline{N_0} + \sum N_i\overline{X_i}\right)\frac{N}{EF}ds + \int\left(\overline{T_0} + \sum T_i\overline{X_i}\right)\frac{T}{GF'}ds +$$

$$+ \int\left(\overline{M_0} + \sum M_i\overline{X_i}\right)\frac{\varepsilon\Delta t}{h}ds + + \int\left(\overline{N_0} + \sum N_i\overline{X_i}\right)\varepsilon t_s\, ds$$

$$\delta + \overline{R_0}w + \underset{J_1\,\overline{X}_1}{\cancel{(R_1 w)}}\overline{X_1} + \underset{J_2\,\overline{X}_2}{\cancel{(R_2 w)}}\overline{X_2} + \cdots + \underset{J_n\,\overline{X}_n}{\cancel{(R_n w)}}\overline{X_n} = \int \overline{M_0}\frac{M}{EI}ds + \int \overline{N_0}\frac{N}{EF}ds + \int \overline{T_0}\frac{T}{GF'}ds +$$

$$+ \int \overline{M_0}\frac{\varepsilon\Delta t}{h}ds + \int \overline{N_0}\varepsilon t_s ds + \cancel{(}\int M_1\frac{M}{EI}ds + \int N_1\frac{N}{EF}ds + \int T_1\frac{T}{GF'}ds + \int M_1\frac{\varepsilon\Delta t}{h}ds + \int N_1\varepsilon t_s ds)\overline{X_1} +$$

$$+\cancel{(}\int M_2\frac{M}{EI}ds + \int N_2\frac{N}{EF}ds + \int T_2\frac{T}{GF'}ds + \int M_2\frac{\varepsilon\Delta t}{d}ds + \int N_2\varepsilon t_s ds)\overline{X_2} + \cdots +$$

$$+\cancel{(}\int M_n\frac{M}{EI}ds + \int N_n\frac{N}{EF}ds + \int T_n\frac{T}{GF'}ds + \int M_n\frac{\varepsilon\Delta t}{d}ds + \int N_n\varepsilon t_s ds)\overline{X_n}$$

$$\boxed{1.\,\delta + \overline{R_0}w = \int \overline{M_0}\frac{M}{EI}ds + \int \overline{N_0}\frac{N}{EF}ds + \int \overline{T_0}\frac{T}{GF'}ds + \int \overline{M_0}\frac{\varepsilon\Delta t}{d}ds + \int \overline{N_0}\varepsilon t_s ds \qquad \textbf{(III)}}$$

Eşitliğin sol ve sağında bulunan parantezli terimler sadeleşiyor.

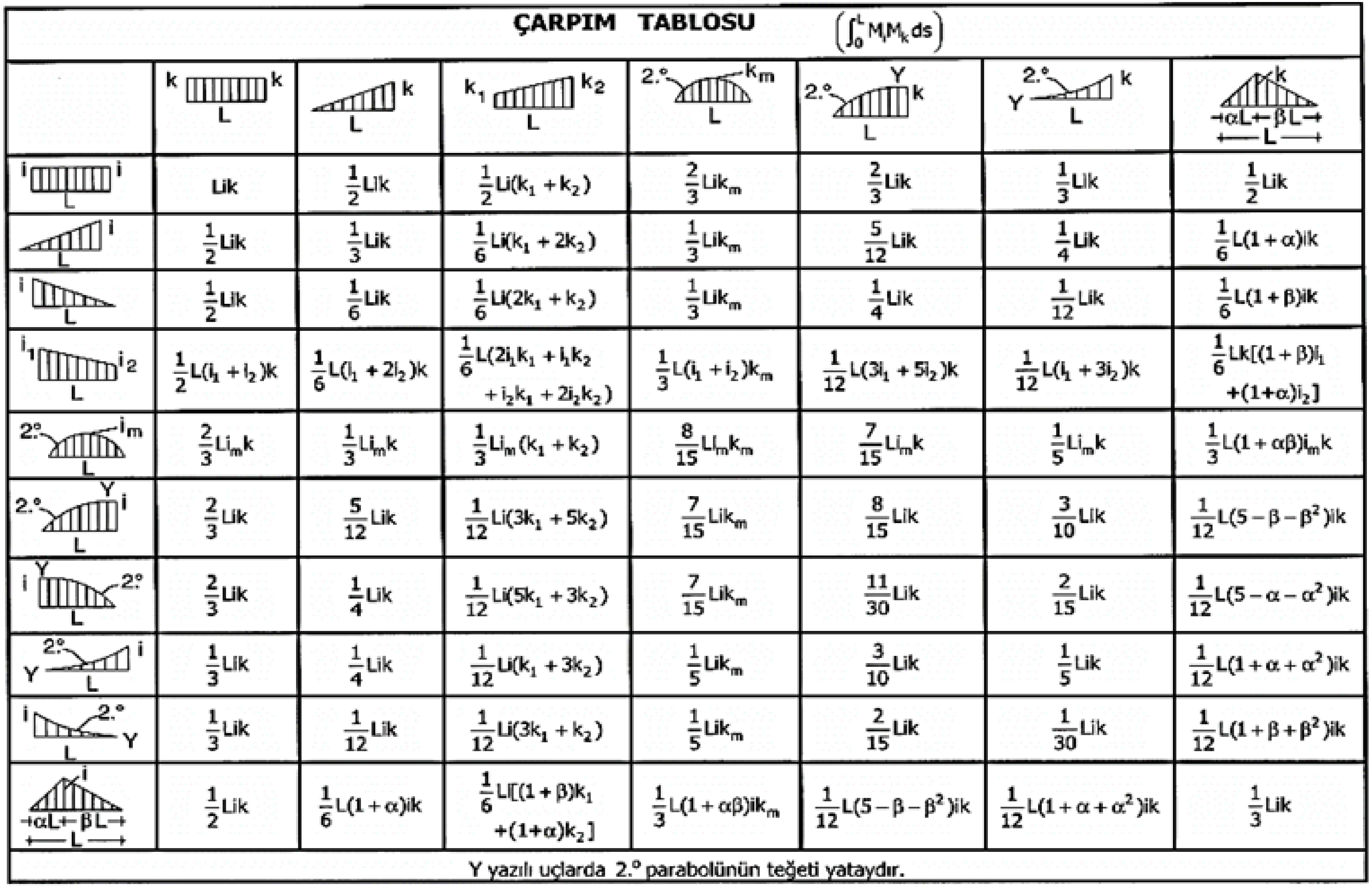
$$\boxed{1.\delta + \overline{R}_0 w = \int \overline{M}_0 \frac{M}{EI} ds + \int \overline{N}_0 \frac{N}{EF} ds + \int \overline{T}_0 \frac{T}{GF'} ds + \int \overline{M}_0 \frac{\varepsilon \Delta t}{d} ds + \int \overline{N}_0 \varepsilon t_s ds \qquad \textbf{(III)}}$$

$$\left.\begin{aligned} M &= M_0 + \sum_{i=1}^{n} M_i X_i \\ N &= N_0 + \sum_{i=1}^{n} N_i X_i \\ T &= T_0 + \sum_{i=1}^{n} T_i X_i \\ R &= R_0 + \sum_{i=1}^{n} R_i X_i \end{aligned}\right\} \textbf{(IV)}$$

(IV) denklemleri (I) de yerine konulursa

$$\boxed{1.\delta + R_0 w = \int \overline{M} \frac{M_0}{EI} ds + \int \overline{N} \frac{N_0}{EF} ds + \int \overline{T} \frac{T_0}{GF'} ds + \int \overline{M} \frac{\varepsilon \Delta t}{d} ds + \int \overline{N} \varepsilon t_s ds \qquad \textbf{(VI)}}$$

(III) denklemi ile (VI) denklemi karşılaştırılırsa = Kısaltma Teoremi √

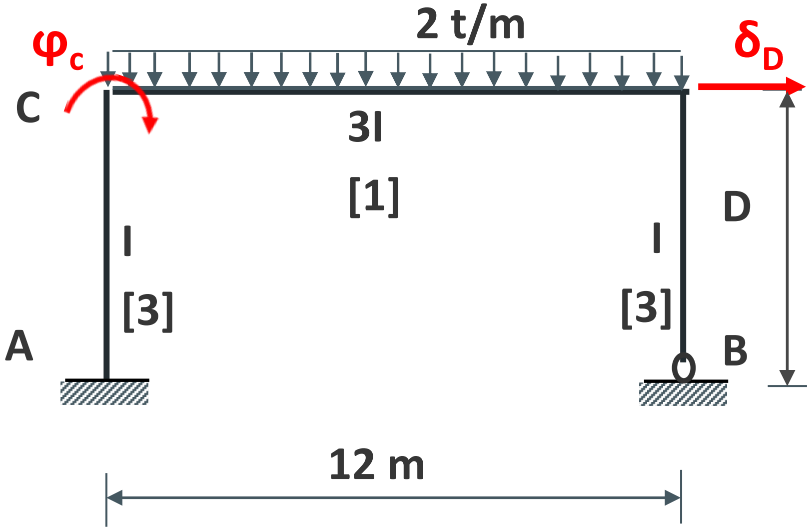


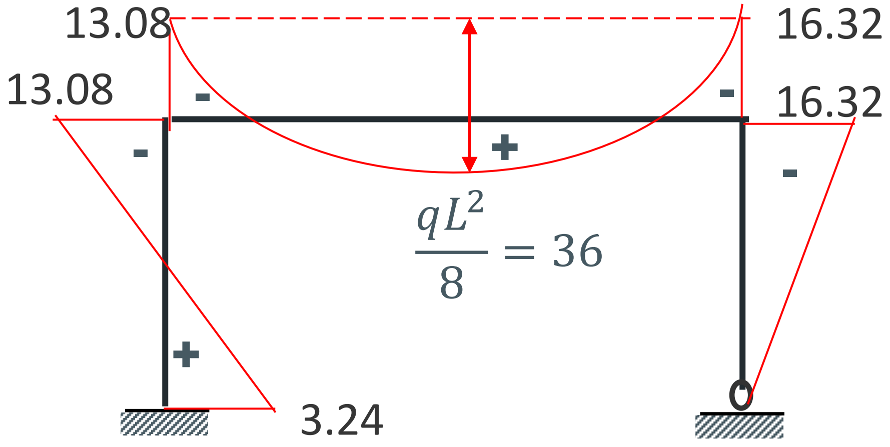
ÇARPIM TABLOSU $\left(\int_0^L M_i M_k ds\right)$

	k k L	k L	k_1 k_2 L	2.° k_m L	2.° Y k L	2.° Y k L	k αL βL L
i i L	Lik	$\frac{1}{2}Lik$	$\frac{1}{2}Li(k_1+k_2)$	$\frac{2}{3}Lik_m$	$\frac{2}{3}Lik$	$\frac{1}{3}Lik$	$\frac{1}{2}Lik$
i L	$\frac{1}{2}Lik$	$\frac{1}{3}Lik$	$\frac{1}{6}Li(k_1+2k_2)$	$\frac{1}{3}Lik_m$	$\frac{5}{12}Lik$	$\frac{1}{4}Lik$	$\frac{1}{6}L(1+\alpha)ik$
i L	$\frac{1}{2}Lik$	$\frac{1}{6}Lik$	$\frac{1}{6}Li(2k_1+k_2)$	$\frac{1}{3}Lik_m$	$\frac{1}{4}Lik$	$\frac{1}{12}Lik$	$\frac{1}{6}L(1+\beta)ik$
i_1 i_2 L	$\frac{1}{2}L(i_1+i_2)k$	$\frac{1}{6}L(i_1+2i_2)k$	$\frac{1}{6}L(2i_1k_1+i_1k_2$ $+i_2k_1+2i_2k_2)$	$\frac{1}{3}L(i_1+i_2)k_m$	$\frac{1}{12}L(3i_1+5i_2)k$	$\frac{1}{12}L(i_1+3i_2)k$	$\frac{1}{6}Lk[(1+\beta)i_1$ $+(1+\alpha)i_2]$
2.° i_m L	$\frac{2}{3}Li_mk$	$\frac{1}{3}Li_mk$	$\frac{1}{3}Li_m(k_1+k_2)$	$\frac{8}{15}Li_mk_m$	$\frac{7}{15}Li_mk$	$\frac{1}{5}Li_mk$	$\frac{1}{3}L(1+\alpha\beta)i_mk$
2.° Y i L	$\frac{2}{3}Lik$	$\frac{5}{12}Lik$	$\frac{1}{12}Li(3k_1+5k_2)$	$\frac{7}{15}Lik_m$	$\frac{8}{15}Lik$	$\frac{3}{10}Lik$	$\frac{1}{12}L(5-\beta-\beta^2)ik$
i Y 2.° L	$\frac{2}{3}Lik$	$\frac{1}{4}Lik$	$\frac{1}{12}Li(5k_1+3k_2)$	$\frac{7}{15}Lik_m$	$\frac{11}{30}Lik$	$\frac{2}{15}Lik$	$\frac{1}{12}L(5-\alpha-\alpha^2)ik$
2.° i Y L	$\frac{1}{3}Lik$	$\frac{1}{4}Lik$	$\frac{1}{12}Li(k_1+3k_2)$	$\frac{1}{5}Lik_m$	$\frac{3}{10}Lik$	$\frac{1}{5}Lik$	$\frac{1}{12}L(1+\alpha+\alpha^2)ik$
i 2.° L Y	$\frac{1}{3}Lik$	$\frac{1}{12}Lik$	$\frac{1}{12}Li(3k_1+k_2)$	$\frac{1}{5}Lik_m$	$\frac{2}{15}Lik$	$\frac{1}{30}Lik$	$\frac{1}{12}L(1+\beta+\beta^2)ik$
i αL βL L	$\frac{1}{2}Lik$	$\frac{1}{6}L(1+\alpha)ik$	$\frac{1}{6}Li[(1+\beta)k_1$ $+(1+\alpha)k_2]$	$\frac{1}{3}L(1+\alpha\beta)ik_m$	$\frac{1}{12}L(5-\beta-\beta^2)ik$	$\frac{1}{12}L(1+\alpha+\alpha^2)ik$	$\frac{1}{3}Lik$

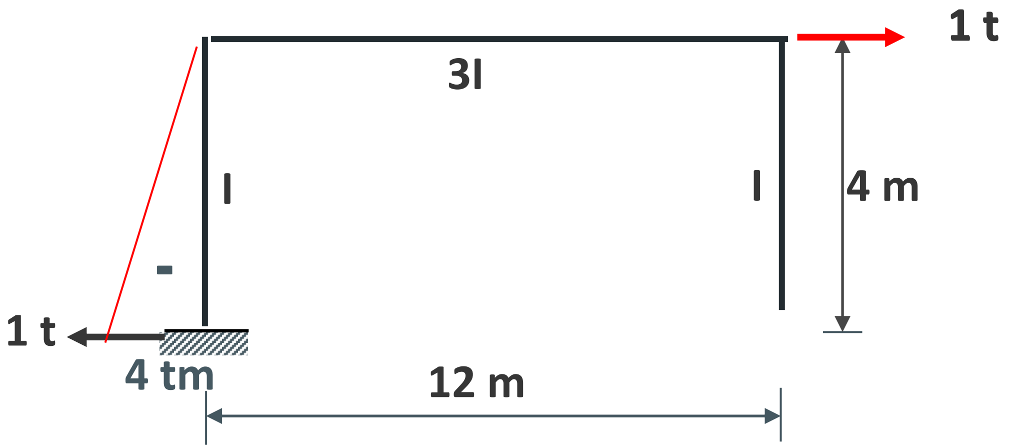
Y yazılı uçlarda 2.° parabolünün teğeti yataydır.

UYGULAMA Dış yük hali

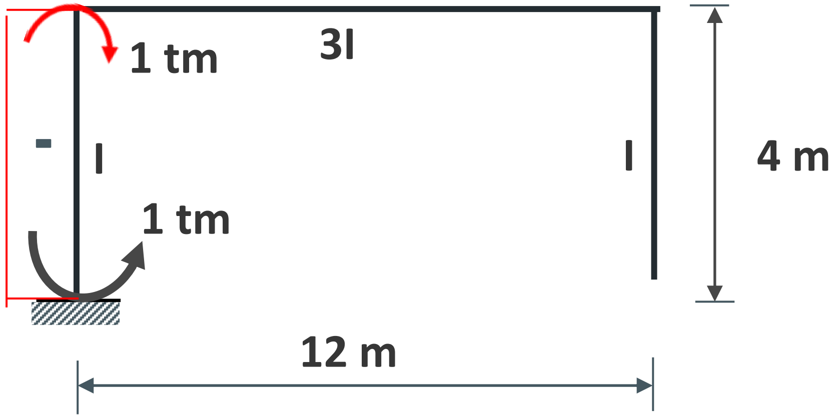




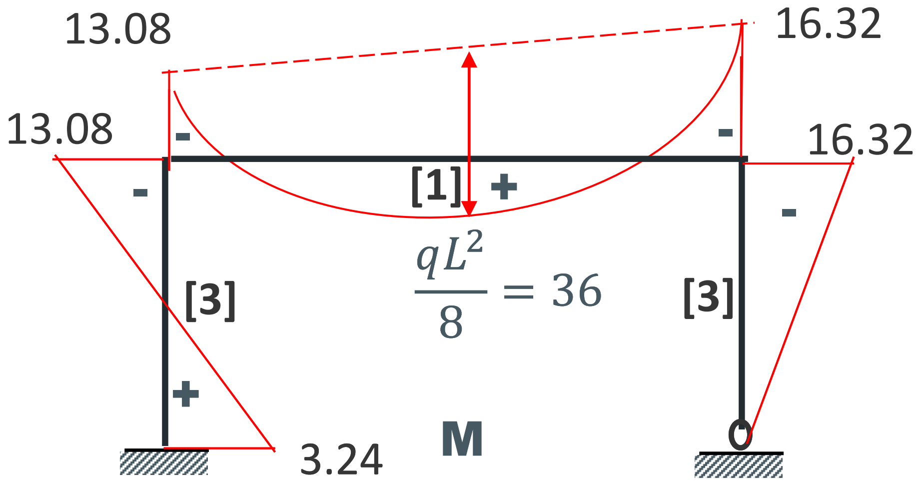
M

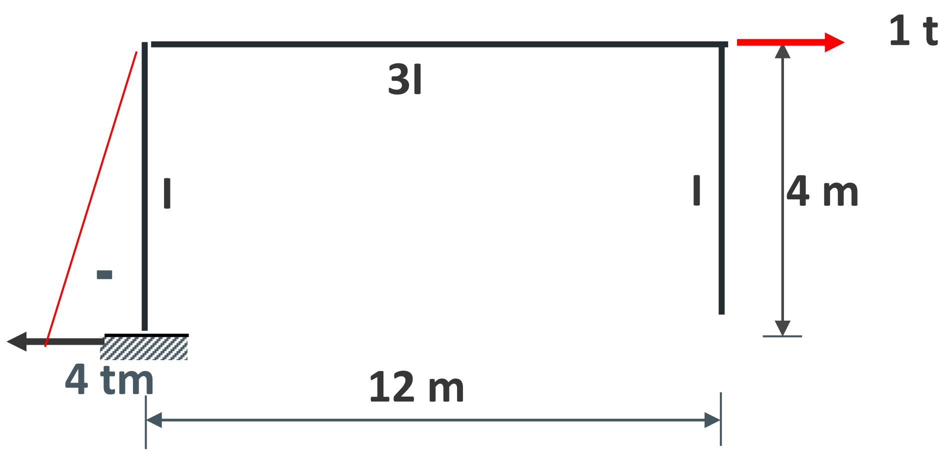


$\overline{M}_0$



$\overline{M}_0$





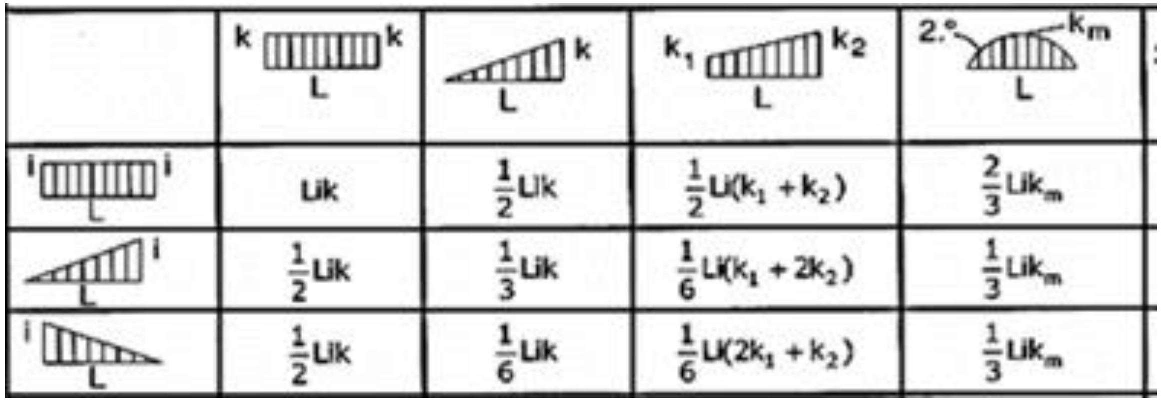
$\overline{M}_0$

$$1 * \delta = \int M\overline{M}\frac{ds}{EI} = \int M\overline{M}_0\frac{ds}{EI} = \int M_0\overline{M}\frac{ds}{EI}$$

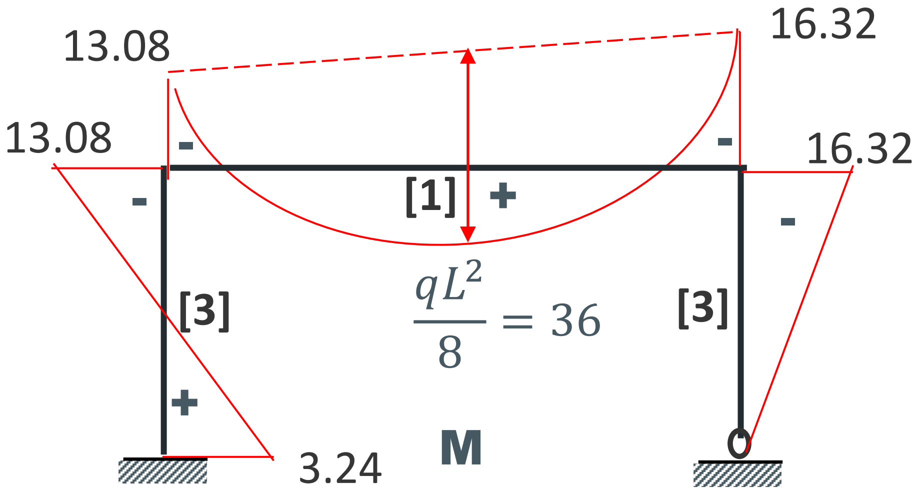
$$EI_c\delta = \int M\,\overline{M}_0[\frac{I_c}{I}]ds$$

$$EI_c\delta_D = \frac{1}{6} * 4 * (-4)(2 * 3.24 - 13.08)[3\} = 52.08$$

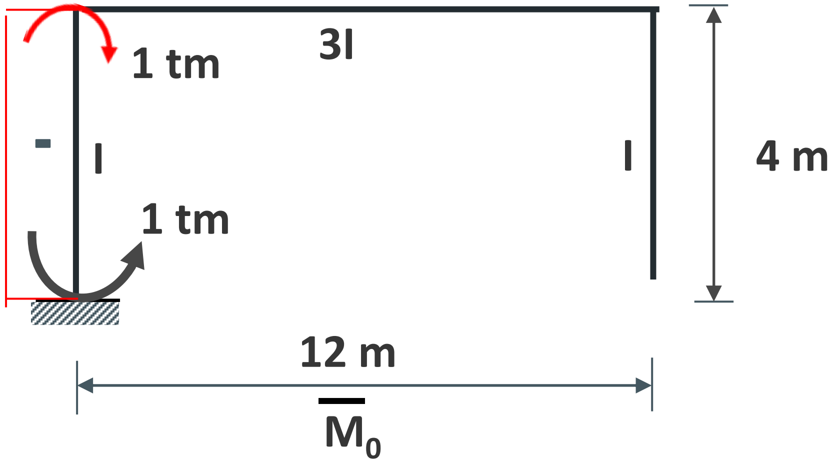
$$\to \delta_D = \frac{52.8}{E3I} = \frac{17.6}{EI}\ m$$



	k k L	L k	k₁ k₂ L	2.° kₘ L
i i L	Lik	$\frac{1}{2}Lik$	$\frac{1}{2}L(k_1+k_2)$	$\frac{2}{3}Lik_m$
i L	$\frac{1}{2}Lik$	$\frac{1}{3}Lik$	$\frac{1}{6}L(k_1+2k_2)$	$\frac{1}{3}Lik_m$
i L	$\frac{1}{2}Lik$	$\frac{1}{6}Lik$	$\frac{1}{6}L(2k_1+k_2)$	$\frac{1}{3}Lik_m$



$$\frac{qL^2}{8} = 36$$

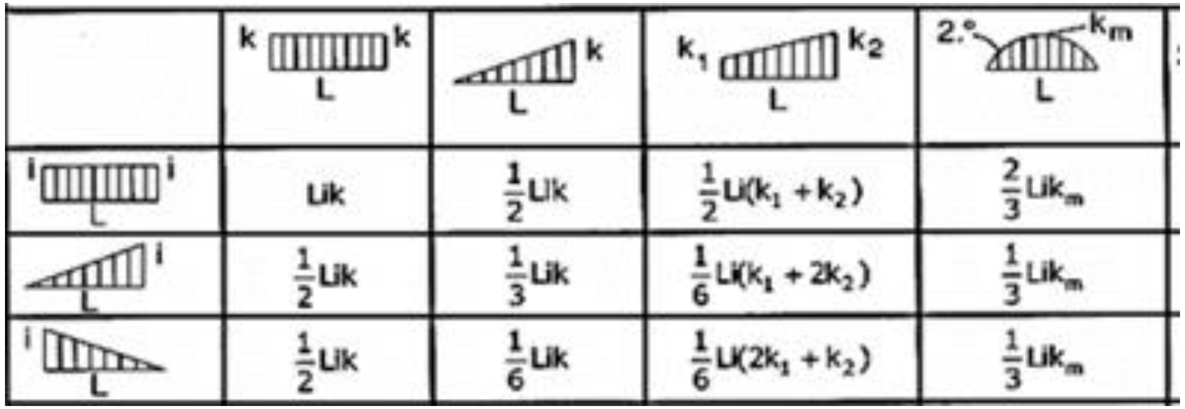


$$1 * \varphi = \int M\overline{M}\frac{ds}{EI} = \int M\overline{M_0}\frac{ds}{EI} = \int M_0\overline{M}\frac{ds}{EI}$$

$$EI_c\varphi_C = \int M\,\overline{M_0}\,\frac{I_c}{I}\,ds$$

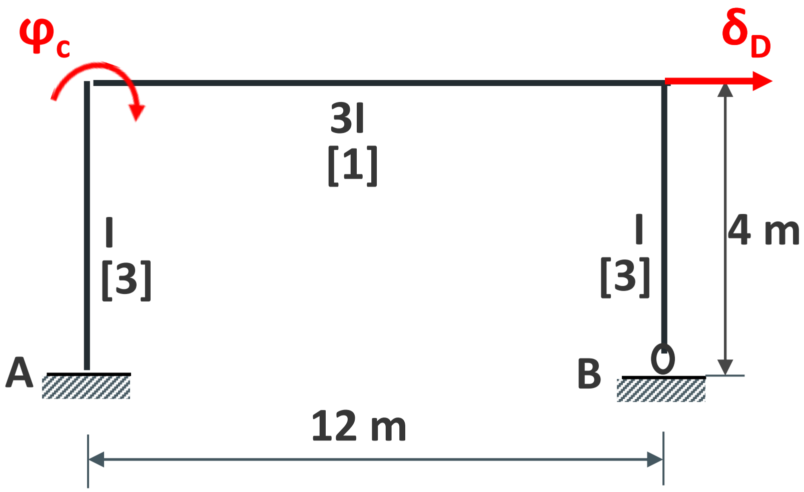
$$EI_c\varphi_C = \frac{1}{2} * 4 * 1 * (3.24 - 1308)[3\} = 59.08$$

$$\rightarrow \varphi_C = \frac{59.08}{E3I} = \frac{19.68}{EI}\ rd$$



	k k L	k L	k₁ k₂ L	2° kₘ L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$L(k₁ + k₂)	$\frac{2}{3}$Likₘ
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$L(k₁ + 2k₂)	$\frac{1}{3}$Likₘ
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$L(2k₁ + k₂)	$\frac{1}{3}$Likₘ

UYGULAMA Sıcaklık değişmesi hali hali (üniform sıcaklık)

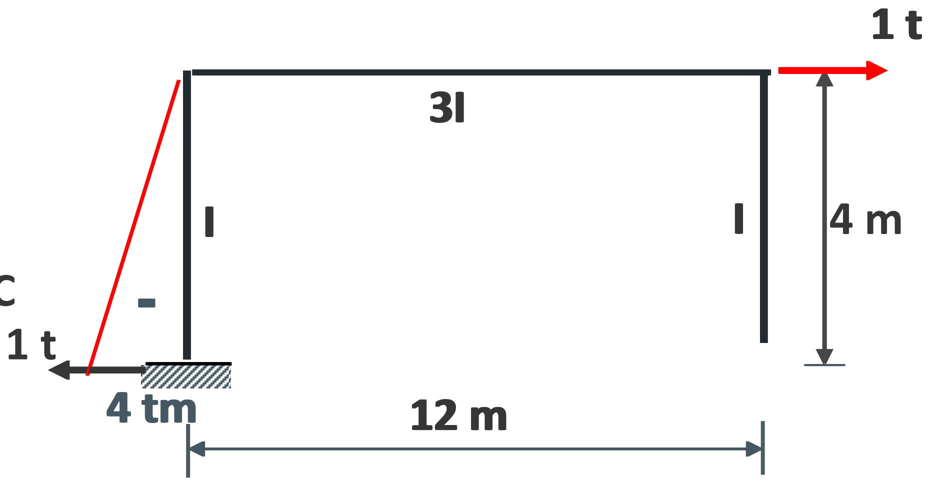


$E=2.1*10^6$ t/m^2

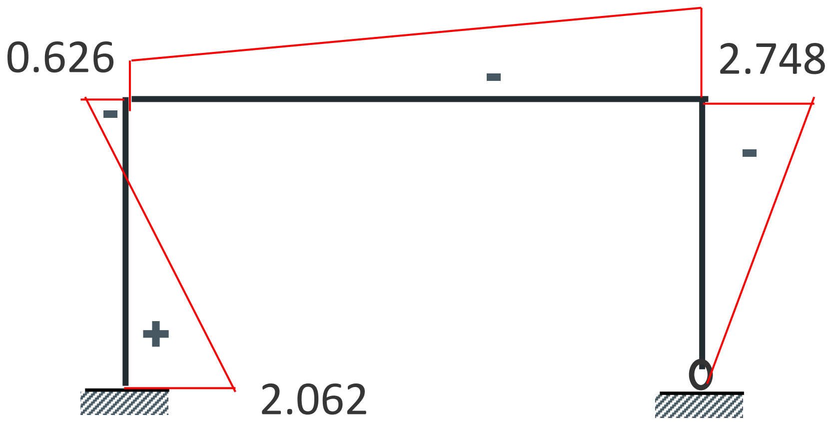
$\varepsilon=10^{-5}$

$I=80$ dm^4

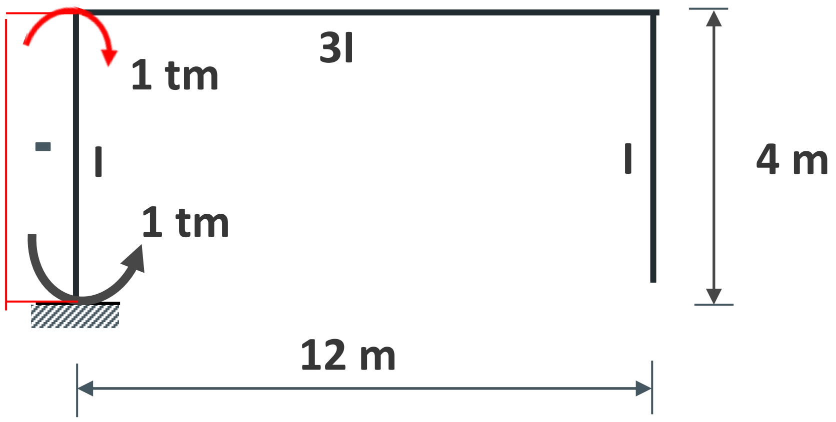
$t_a = t_ü = t_s = 20$ °C



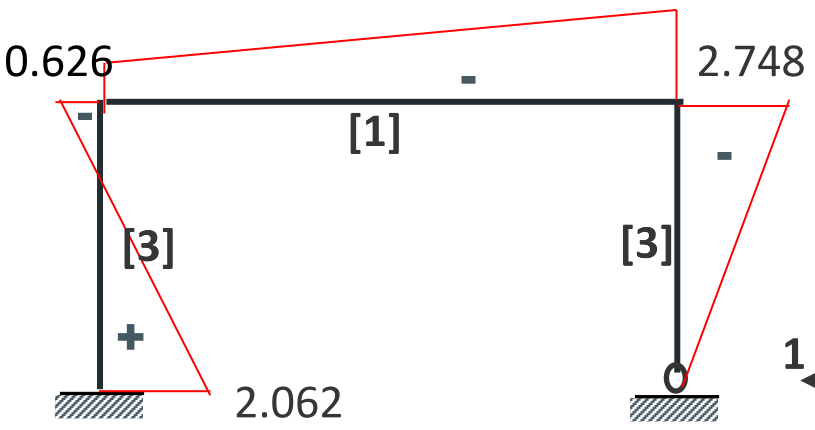
$\overline{M}_0$



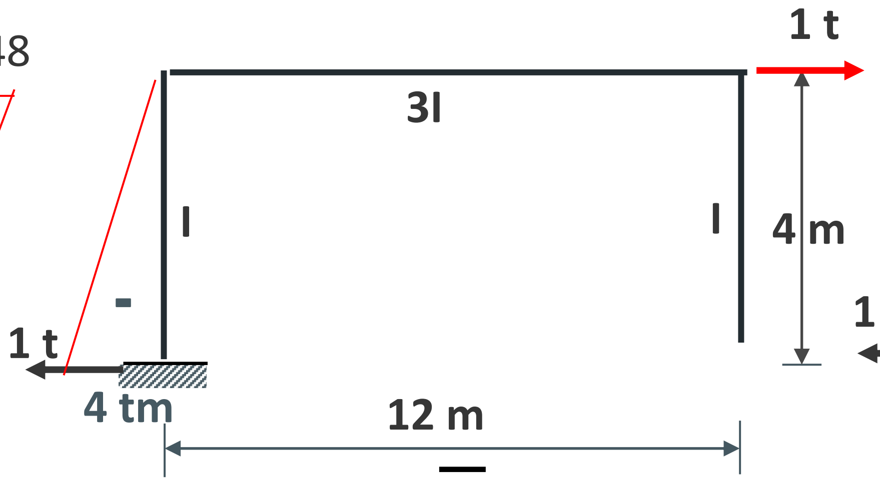
M



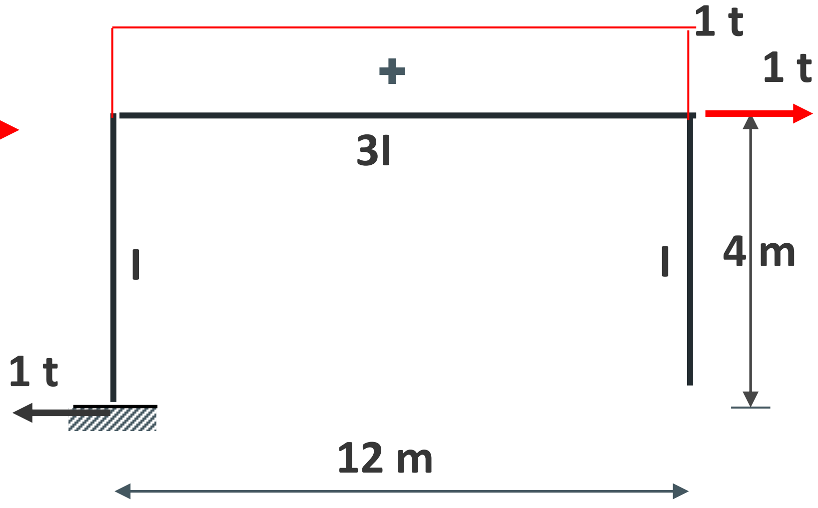
$\overline{M}_0$



M

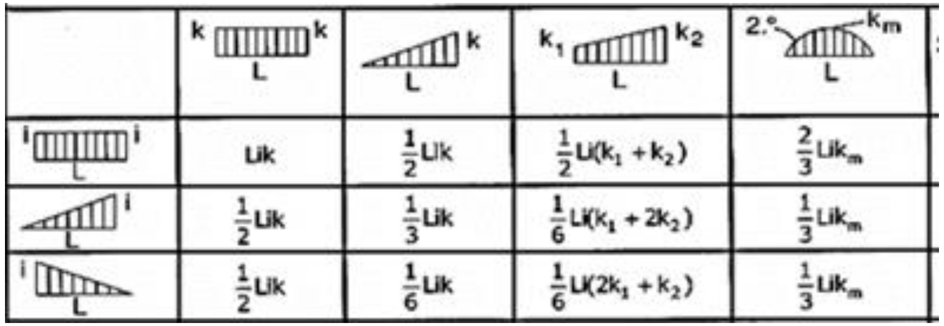


$\bar{\mathbf{M}}_0$



$\bar{\mathbf{N}}_0$

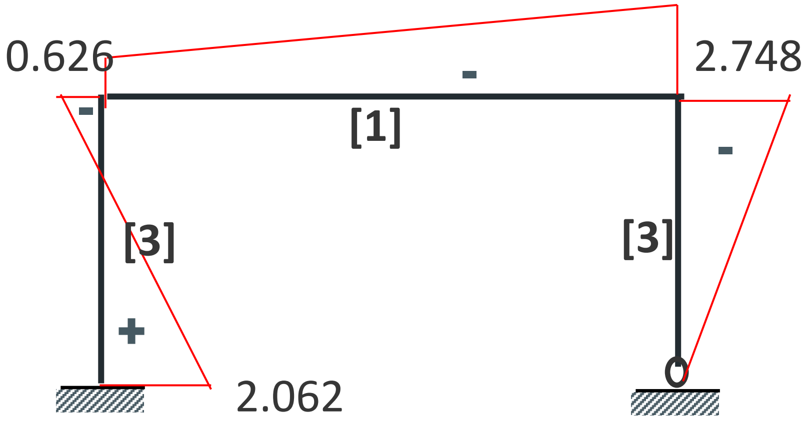
$$EI_c\delta = \int M\,\bar{M}_0 \frac{I_c}{I} ds + EI_c \int \bar{N}_0 \varepsilon t_s ds$$



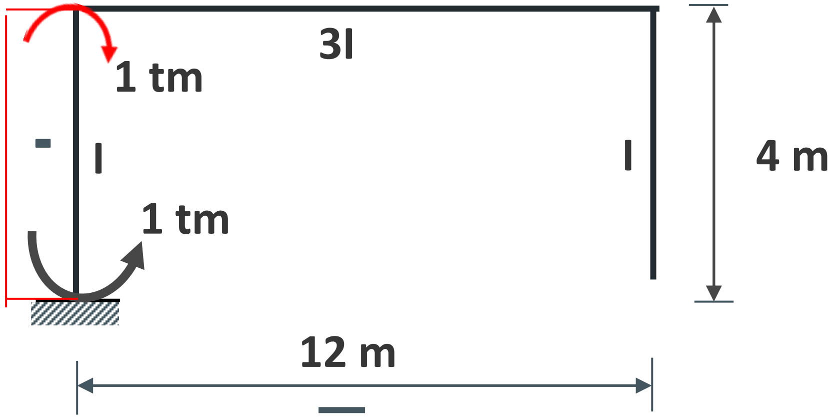
	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1 + k_2)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1 + 2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k_1 + k_2)	$\frac{1}{3}$Lik$_m$

$$EI_c\delta_D = \frac{1}{6} * 4 * (-4)(2 * 2.062 - 0.626)[3\} + 12 * 1 * EI_c\varepsilon t_s = -27.98 + 12EI_c\varepsilon t_s$$

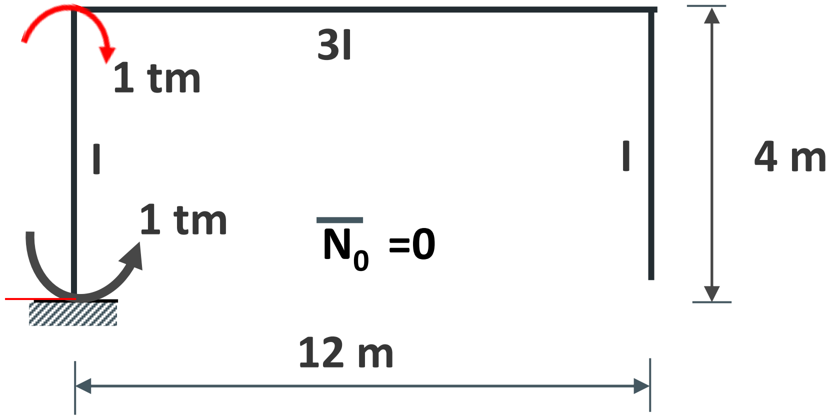
$$\rightarrow \delta_D = -\frac{27.98}{EI_c} + 12\varepsilon t_s$$



M



$\overline{\mathbf{M}}_0$

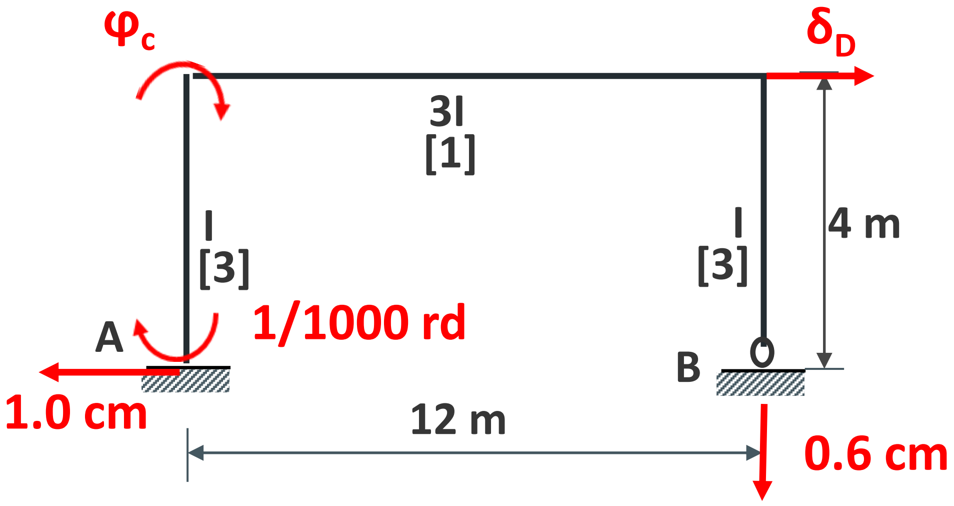


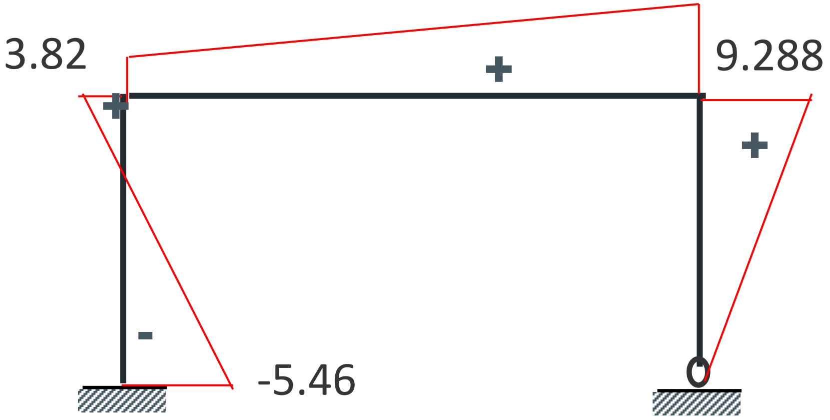
$$EI_c\varphi = \int M\,\bar{M}_0 \frac{I_c}{I} ds + EI_c \int \bar{N}_0 \varepsilon t_s ds$$

$$EI_c\varphi_C = \frac{1}{2} * 4 * (-1) * (2 * 2.062 - 0.626)[3\} + 0 = -20.988$$

$$\rightarrow \varphi_C = -\frac{20.988}{E3I} = -\frac{6.996}{EI}\ rd$$

UYGULAMA Mesnet çökmesi hali hali





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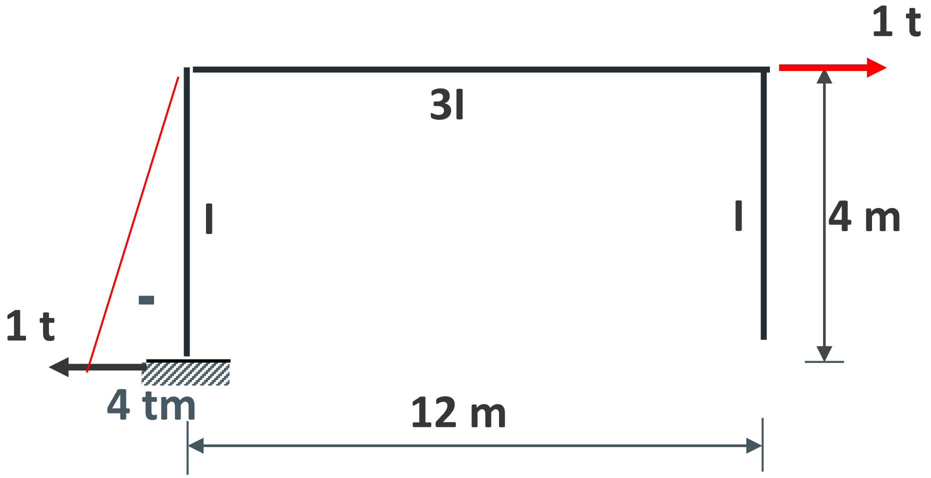
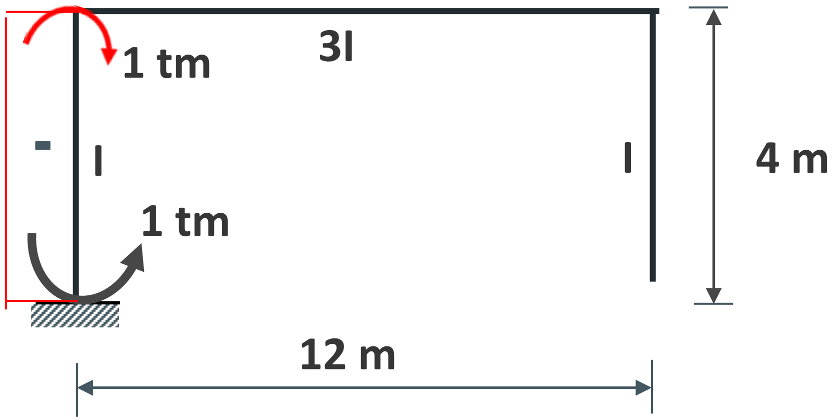
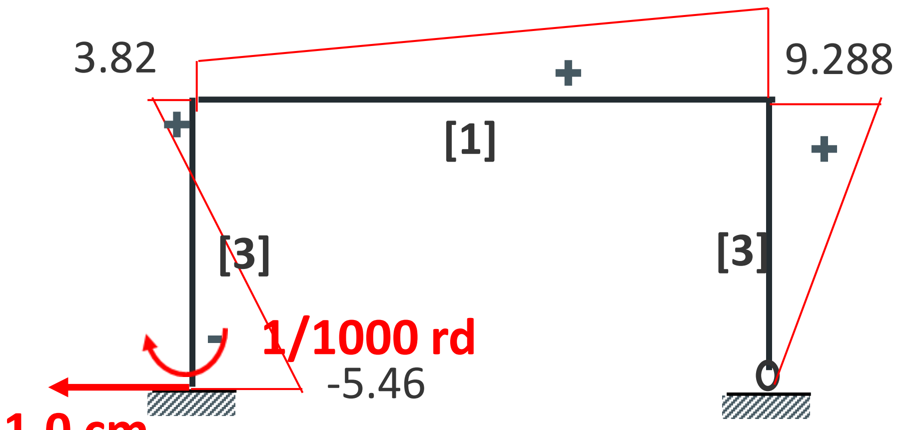


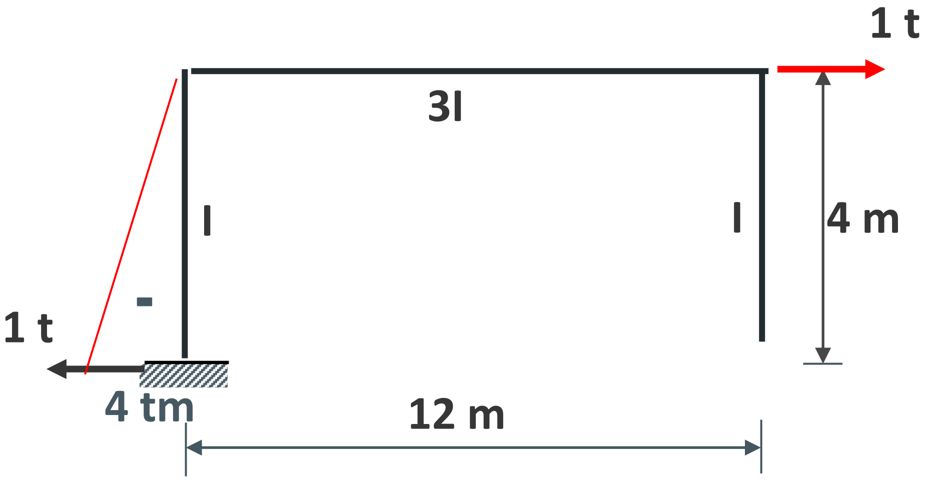
$\overline{M}_0$



$\overline{M}_0$



M



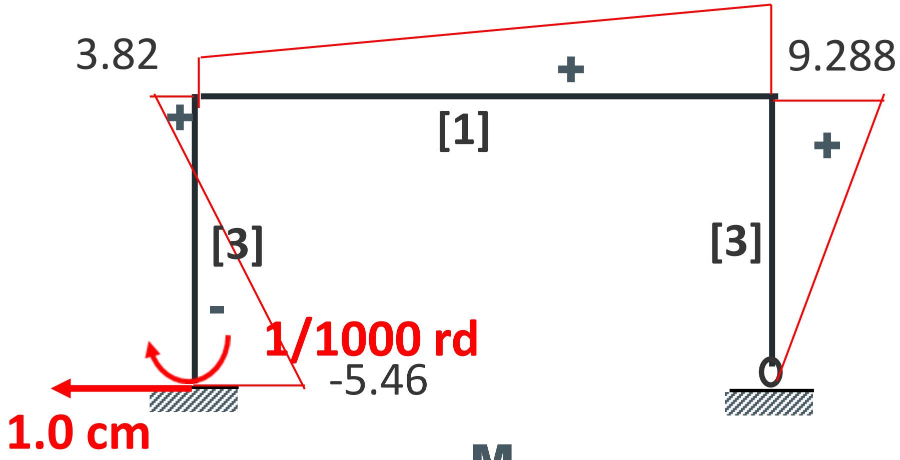
$\bar{M}_0$

$$1 * \delta + \bar{R}_0 w = \int M\bar{M}\frac{ds}{EI} = \int M\bar{M}_0\frac{ds}{EI} = \int M_0\bar{M}\frac{ds}{EI}$$

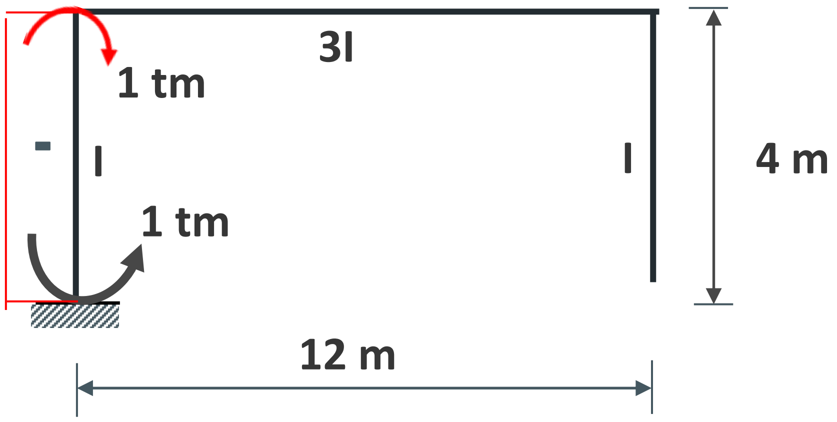
$$EI_c\delta + EI_c\bar{R}_0 w = \int \bar{M}_0 M\left[\frac{I_c}{I}\right]ds$$

$$EI_c\delta_D + EI_c\overbrace{(1 * 0.01 - 4 * \frac{1}{1000})}^{0.006} = \frac{1}{6} * 4 * (-4)(2 * (-5.46) + 3.82)[3\} = 56.8$$

$$\to \delta_D = \frac{56.8}{E3I} - 0.006 = \frac{18.93}{EI} - 0.006\,m$$



M

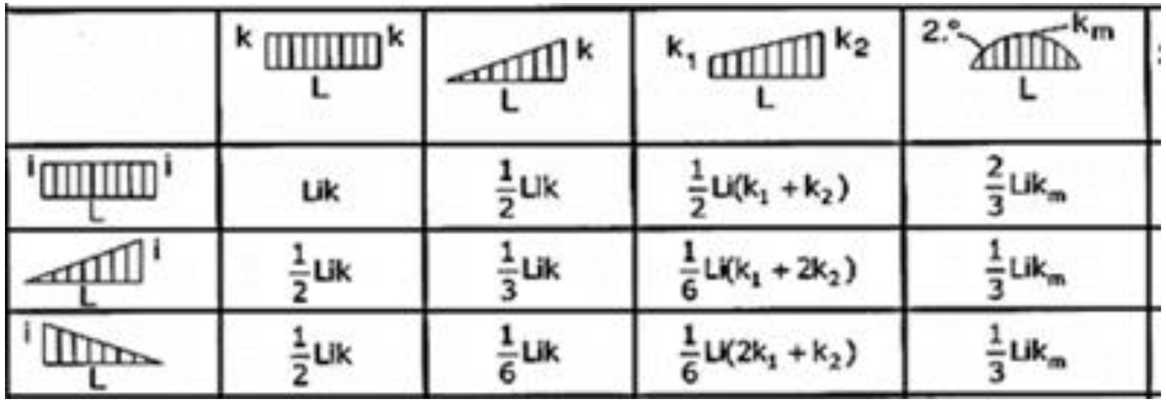


$\bar{M}_0$

$$EI_c\varphi + EI_c\bar{R}_0 w = \int M\bar{M}_0\left[\frac{I_c}{I}\right]ds$$

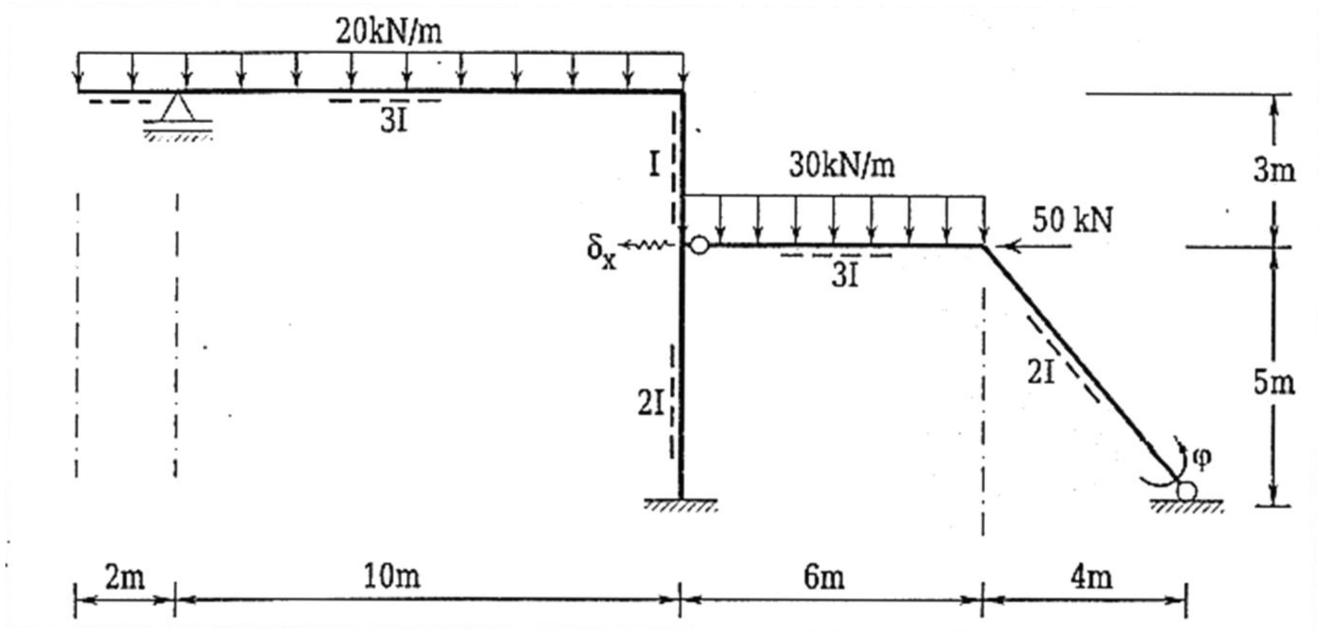
$$EI_c\varphi_C - 1*\frac{1}{1000}EI_c = \frac{1}{2}*4*(-1)*(-5.46+3.82)[3\} = 9.84$$

$$\to \varphi_C = -\frac{9.84}{E3I} + 0.001 = \frac{3.28}{EI} + 0.001\,rd$$



	k k L	k L	k_1 k_2 L	2° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li($2k_1+k_2$)	$\frac{1}{3}$Lik_m

ÖRNEK 10

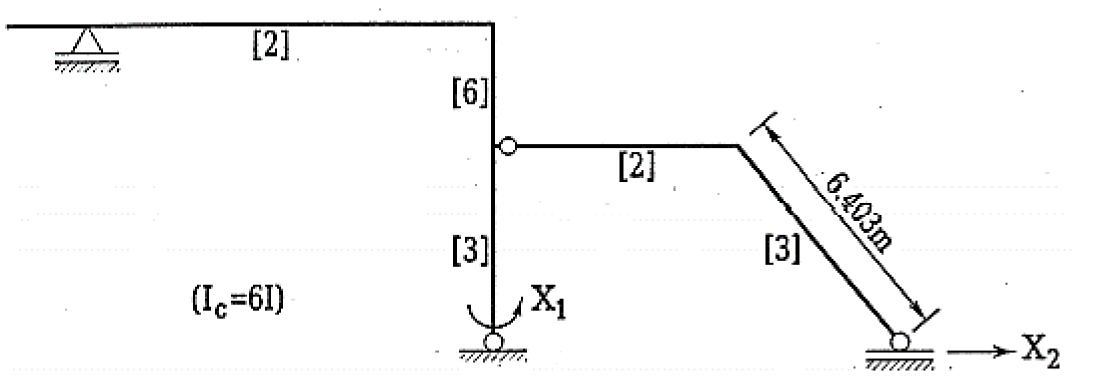


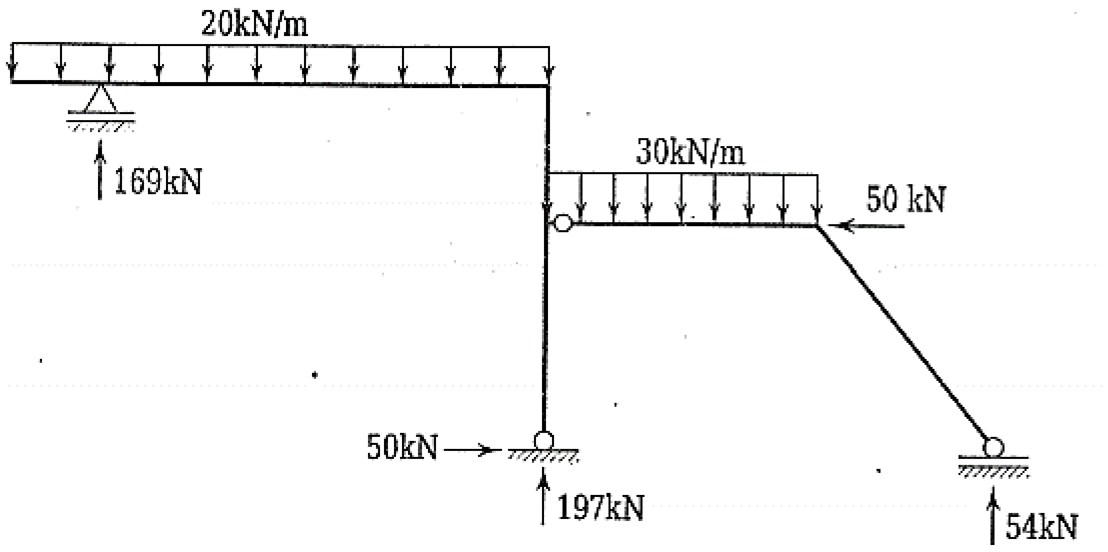
Şekilde verilen sistemin;

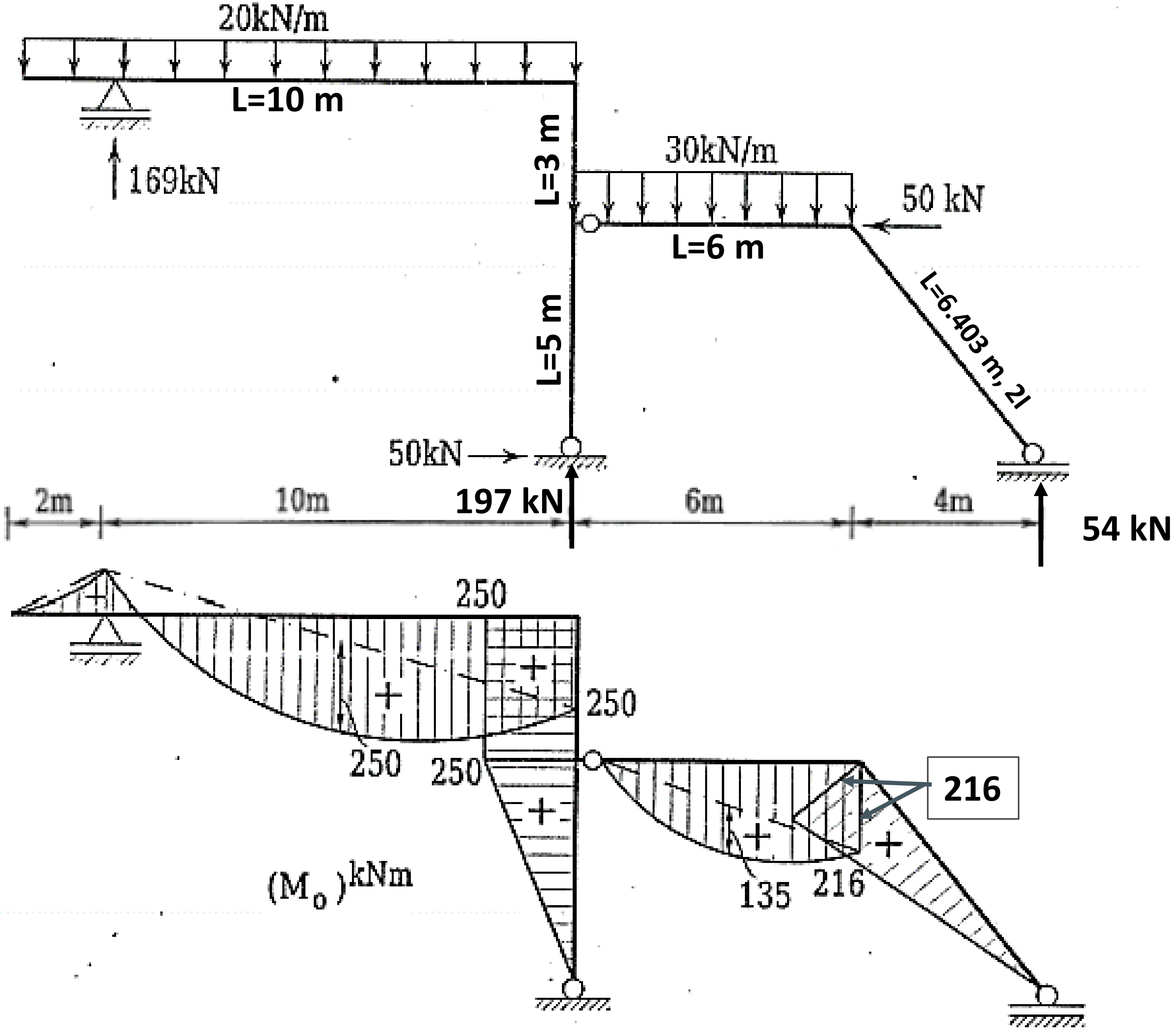
a) Kuvvet yöntemini kullanarak M diyagramını çiziniz. (Sadece eğilme etkisini göz önüne alınız.)

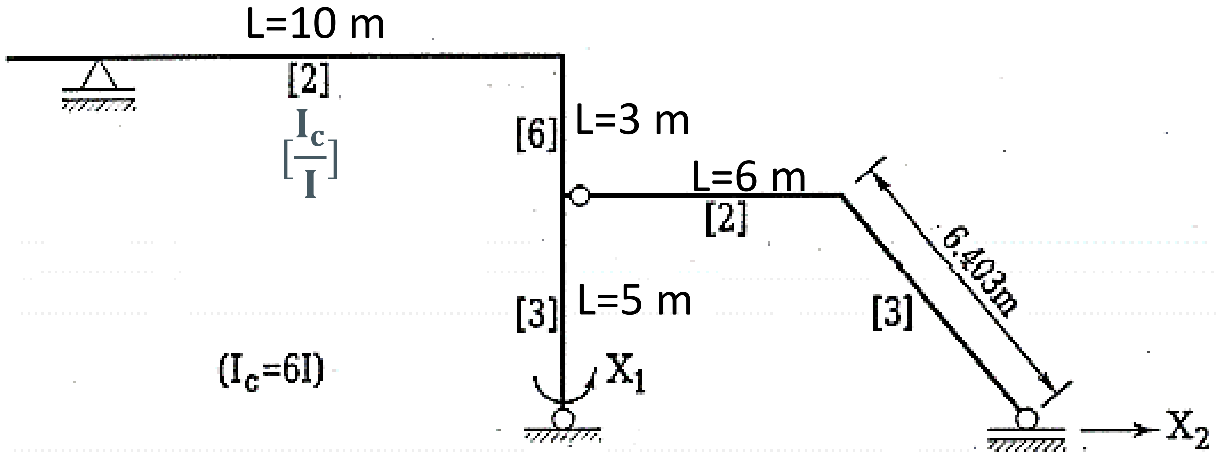
b) δ_x yatay yer değiştirmesini, φ dönmesini hesaplayınız. (EI=90000 kNm^2)

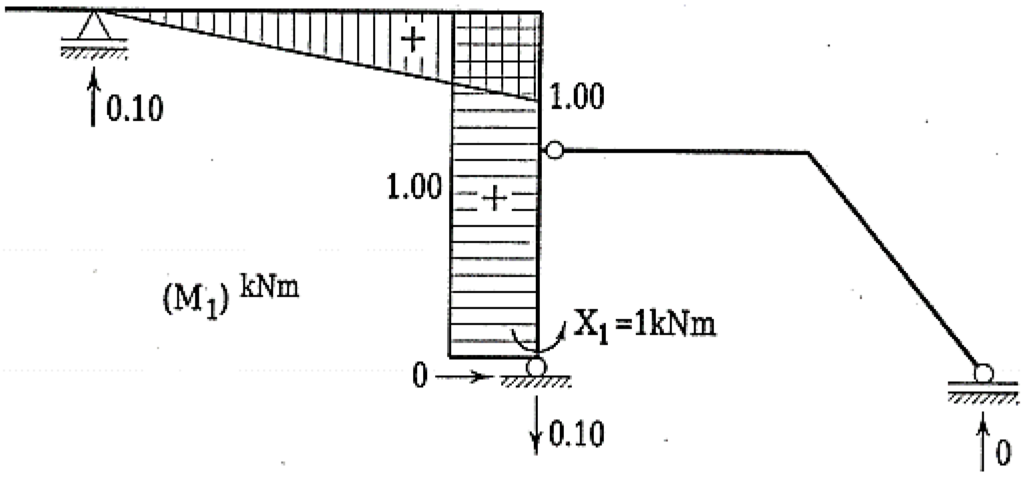
n=3x0+6-1-3=2 2. derece hiperstatik






20kN/m
L=10 m
L=3 m
169kN
30kN/m
50 kN
L=6 m
L=6.403 m, 2I
L=5 m
50kN
197 kN
2m
10m
6m
4m
54 kN
250
250
250
250
216
135
216
(M$_o$)^{kNm}
+



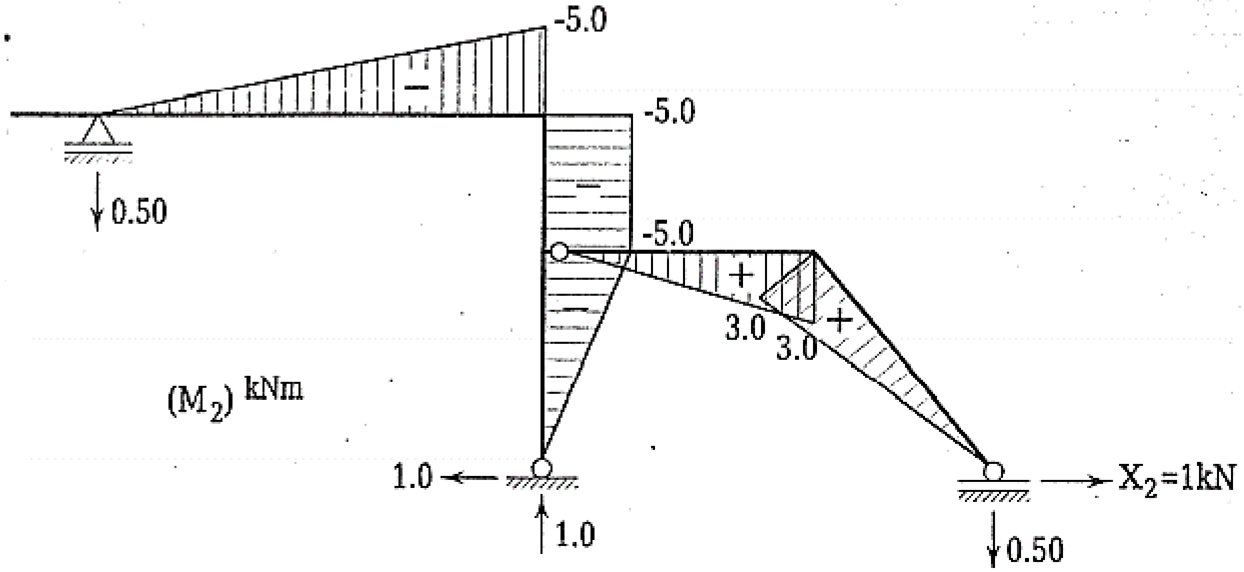


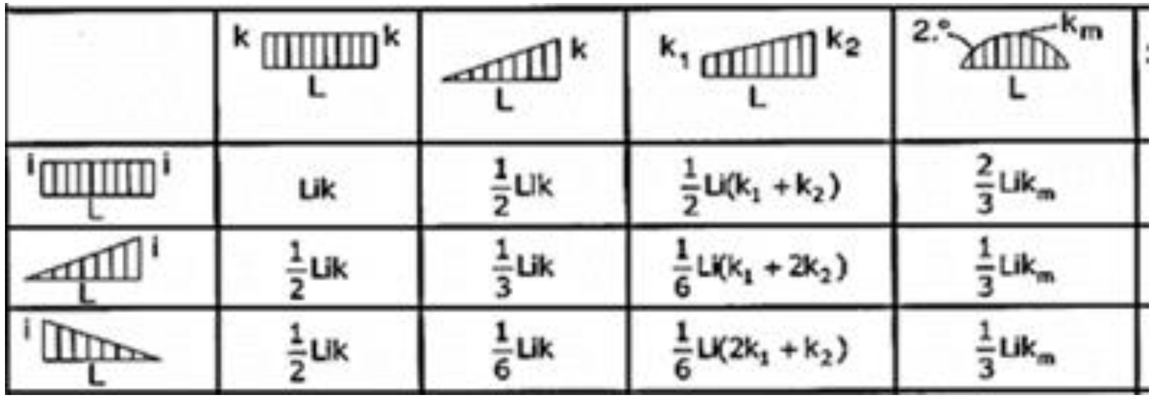
$$EI_c\delta_{11}=\frac{1}{3}\times10\times1\times1\times[2]+1\times1\times3\times[6]+1\times1\times5\times[3]=39.6667$$

$$EI_c\delta_{22}=\frac{1}{3}\times5\times5\times10\times[2]+3\times5\times5\times[6]+\frac{1}{3}\times5\times5\times5\times[3]+\frac{1}{3}\times3\times3\times6\times[2]+\ldots$$

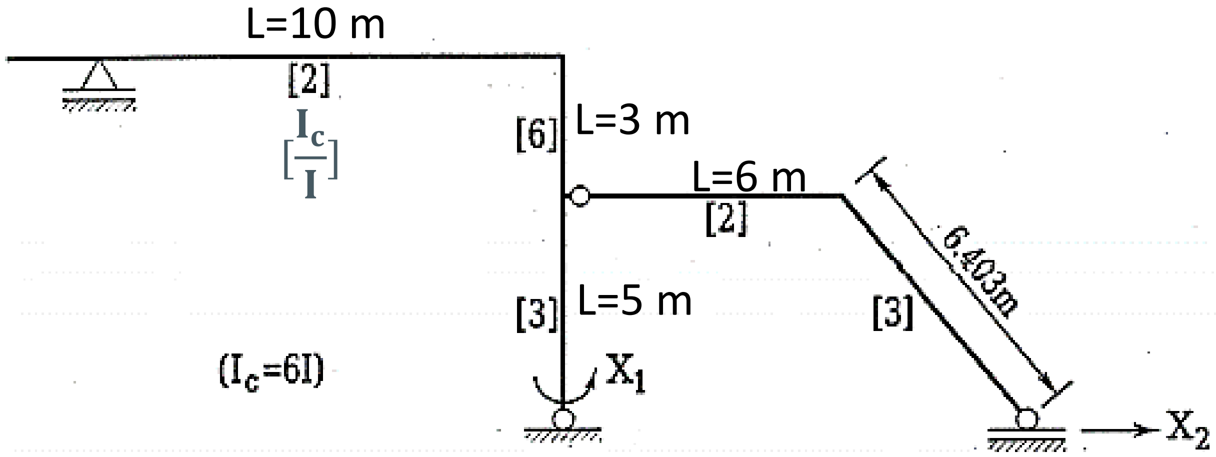
$$\ldots+\frac{1}{3}\times3\times3\times6.403\times[3]=835.2937$$

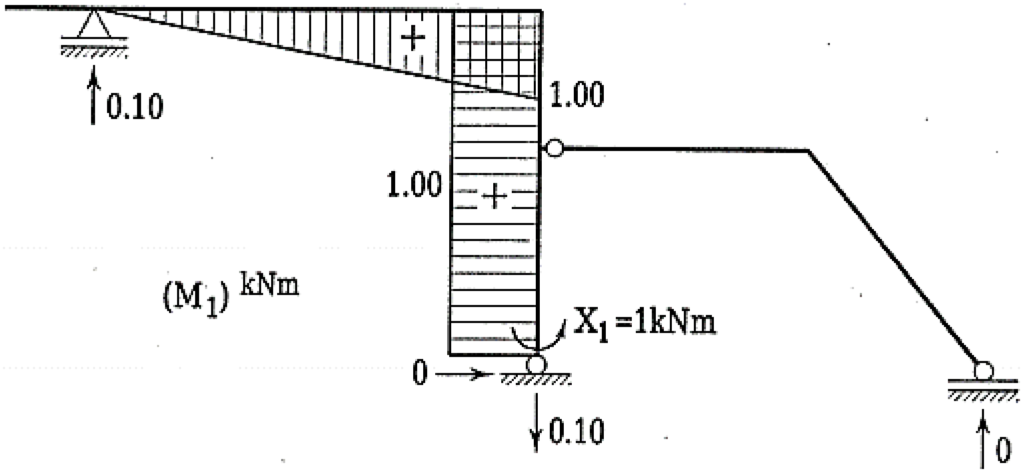
$$EI_c\delta_{12}=\frac{1}{3}\times10\times1\times(-5)\times[2]+1\times(-5)\times3\times[6]+\frac{1}{2}\times5\times1\times(-5)\times[3]=-160.8333$$





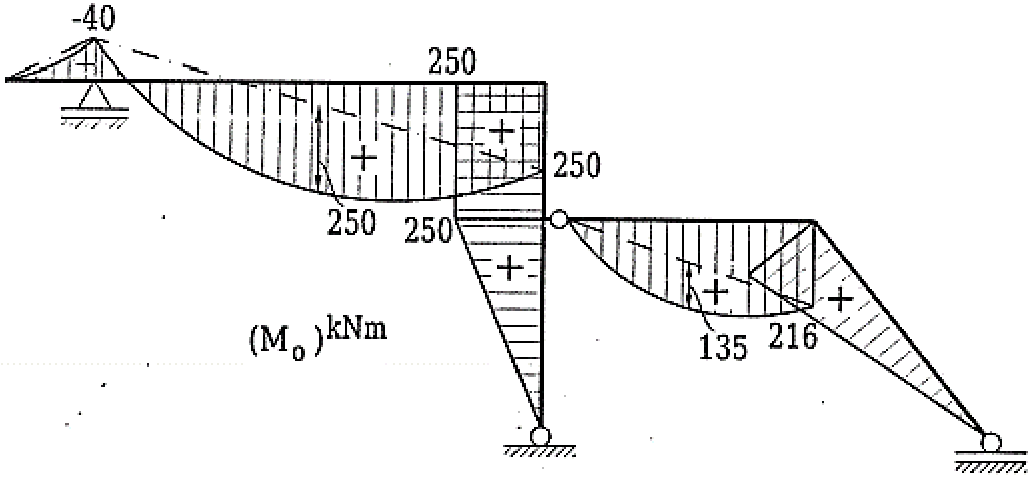
	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$L(k_1+k_2)	$\frac{2}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$L$i(k_1+2k_2)$	$\frac{1}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$L$i(2k_1+k_2)$	$\frac{1}{3}$Lik_m

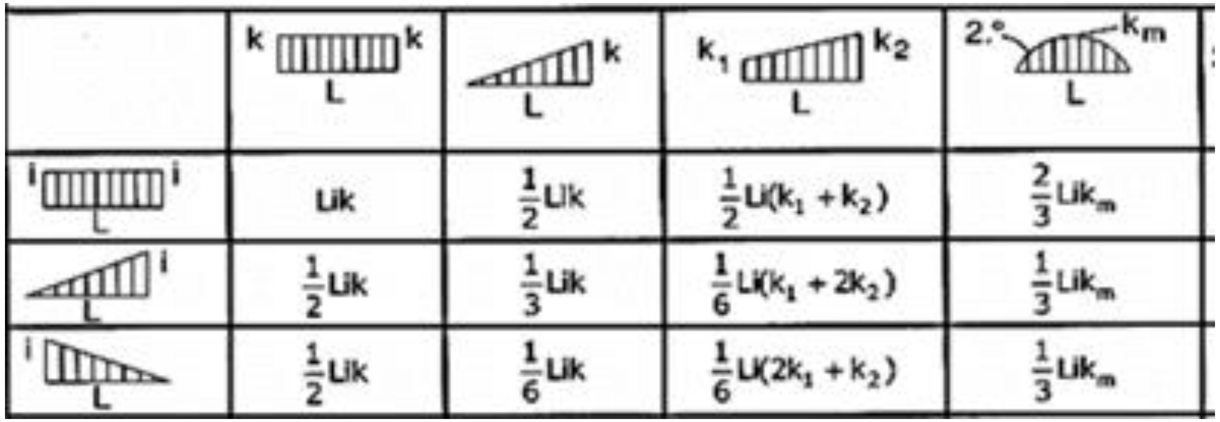




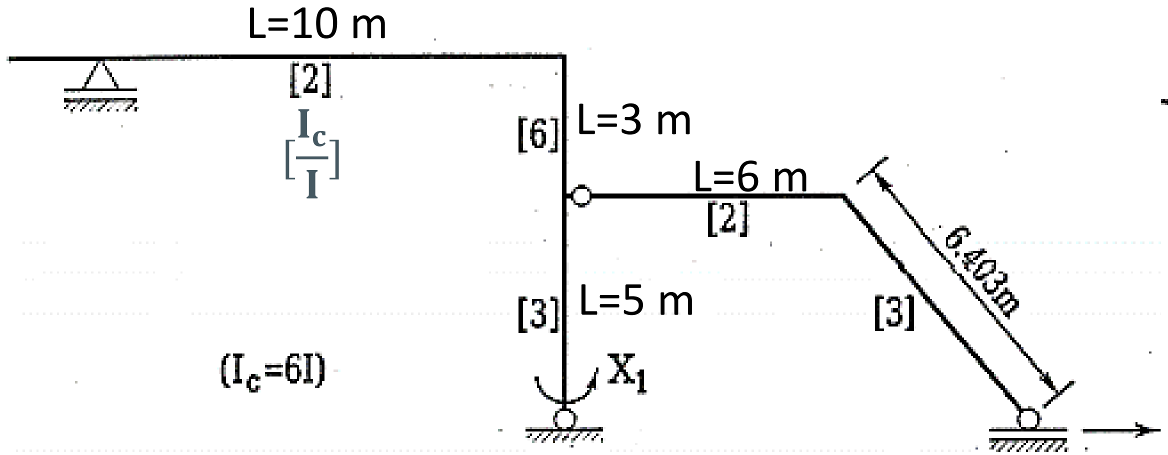
$$EI_c\delta_{10}=\frac{1}{6}\times10\times1\times(2\times250-40)\times[2]+\frac{1}{3}\times10\times1\times250\times[2]+1\times250\times3\times[6]+$$

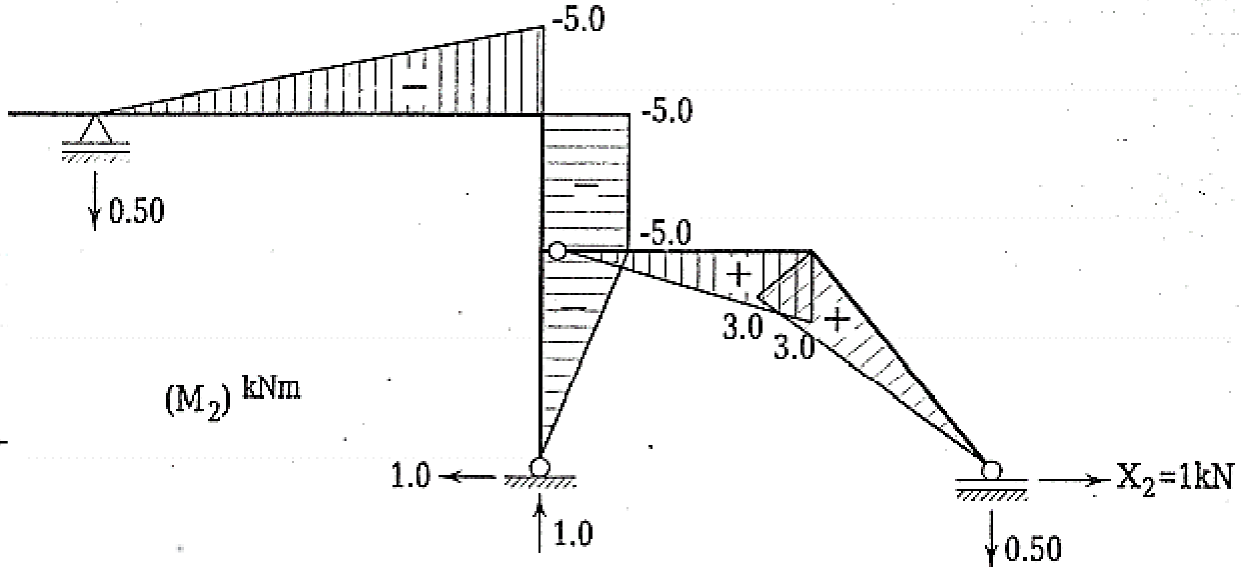
$$...+\frac{1}{2}\times5\times1\times250\times[3]=9575$$





	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li($2k_1+k_2$)	$\frac{1}{3}$Lik_m

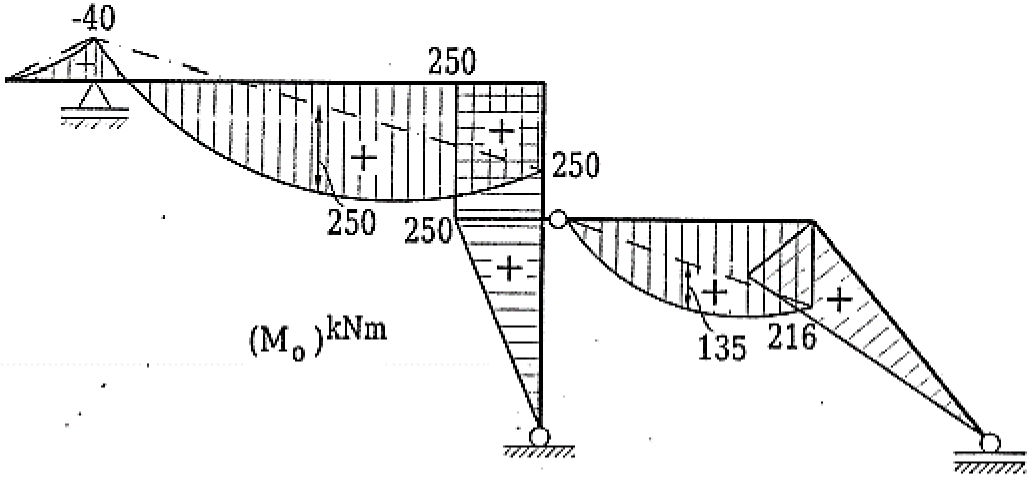


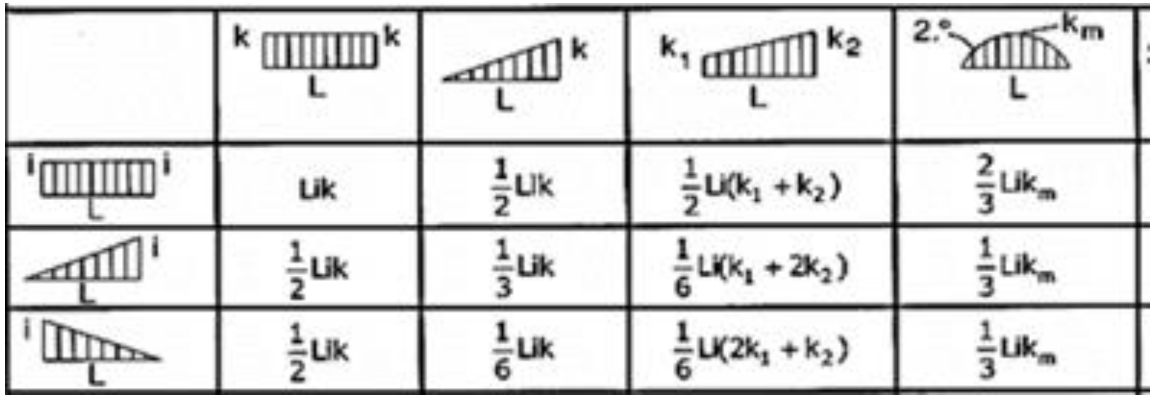


$$EI_c\delta_{20}=\frac{1}{6}\times10\times(-5)\times(2\times250-40)\times[2]+\frac{1}{3}\times10\times(-5)\times250\times[2]+\dots$$

$$\dots+3\times250\times(-5)\times[6]+\frac{1}{3}\times5\times(-5)\times250\times[3]+\frac{1}{3}\times6\times(216)\times3\times[2]+$$

$$\dots+\frac{1}{3}\times6\times(135)\times3\times[2]+\frac{1}{3}\times(6.403)\times(216)\times3\times[3]=-36388.856$$





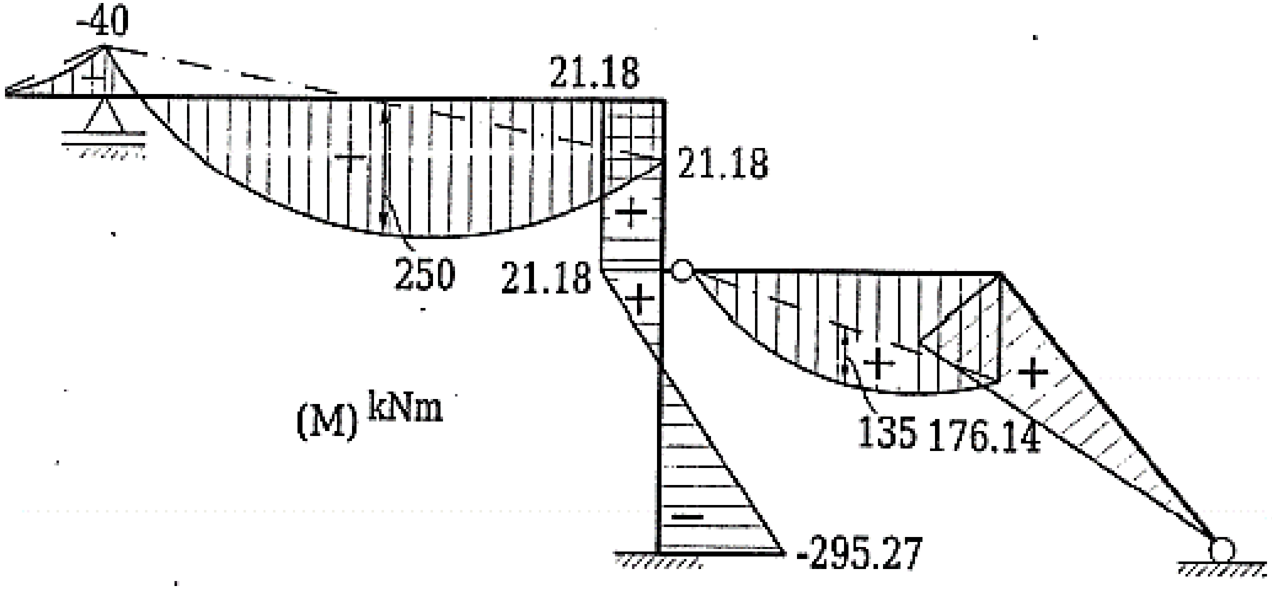
	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li($2k_1+k_2$)	$\frac{1}{3}$Lik$_m$

Hiperstatik bilinmeyenlerin bulunması:

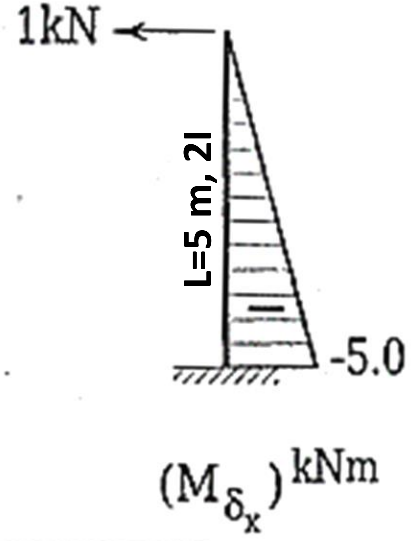
$$\begin{bmatrix} 39.6667 & -160.8333 \\ -160.8333 & 835.2937 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -9575 \\ 36388.856 \end{bmatrix} \Rightarrow X_1 = -295.265 \,,\, X_2 = -13.288$$

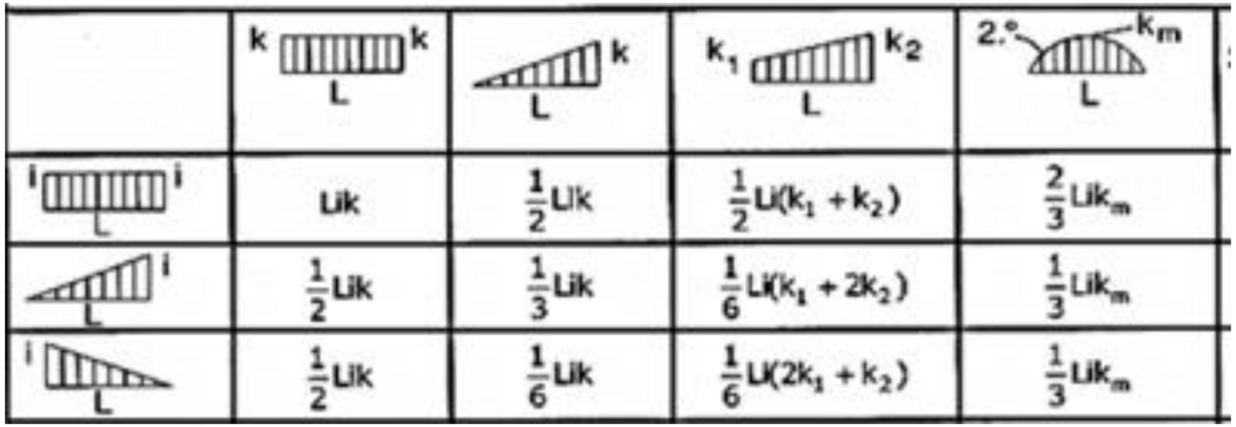
Soruda verilen hiperstatik sisteme ait eğilme momenti diyagramının elde edilmesi:

$$M = M_0 + M_1X_1 + M_2X_2 \quad \Rightarrow \quad M = M_0 - 295.265 \times M_1 - 13.288 \times M_2$$

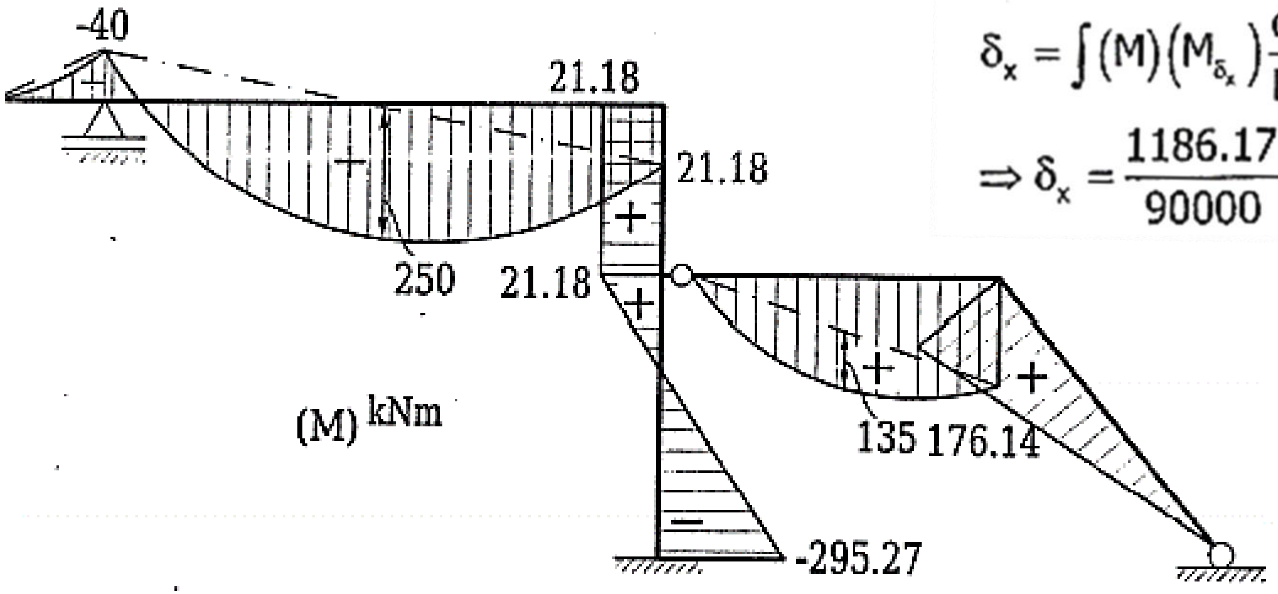


b) $\delta_x = 1$ kN luk birim yüklemeden oluşan eğilme momenti diyagramı





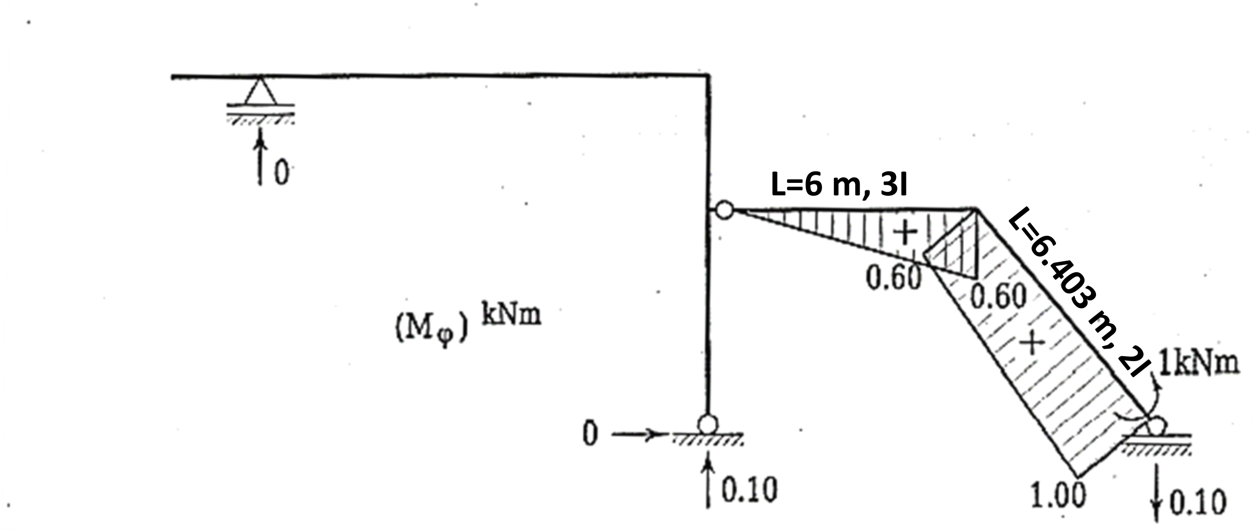
	k ▥ k, L	◺ k, L	k_1 ▥ k_2, L	2.° k_m, L
i ▥ i, L	Lik	$\frac{1}{2}Lik$	$\frac{1}{2}Li(k_1+k_2)$	$\frac{2}{3}Lik_m$
◺ i, L	$\frac{1}{2}Lik$	$\frac{1}{3}Lik$	$\frac{1}{6}Li(k_1+2k_2)$	$\frac{1}{3}Lik_m$
i ◹, L	$\frac{1}{2}Lik$	$\frac{1}{6}Lik$	$\frac{1}{6}Li(2k_1+k_2)$	$\frac{1}{3}Lik_m$

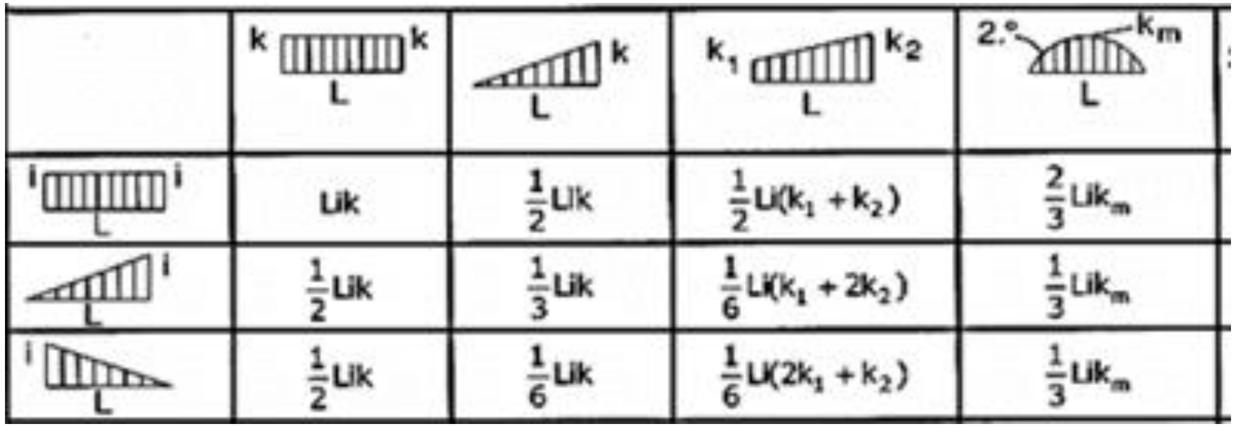


$$\delta_x = \int (M)(M_{\delta_x})\frac{ds}{EI} = \frac{1}{6}\times 5\times(-5)\times(2\times(-295.27)+21.18)\times\frac{1}{2EI} = \frac{1186.17}{EI}$$

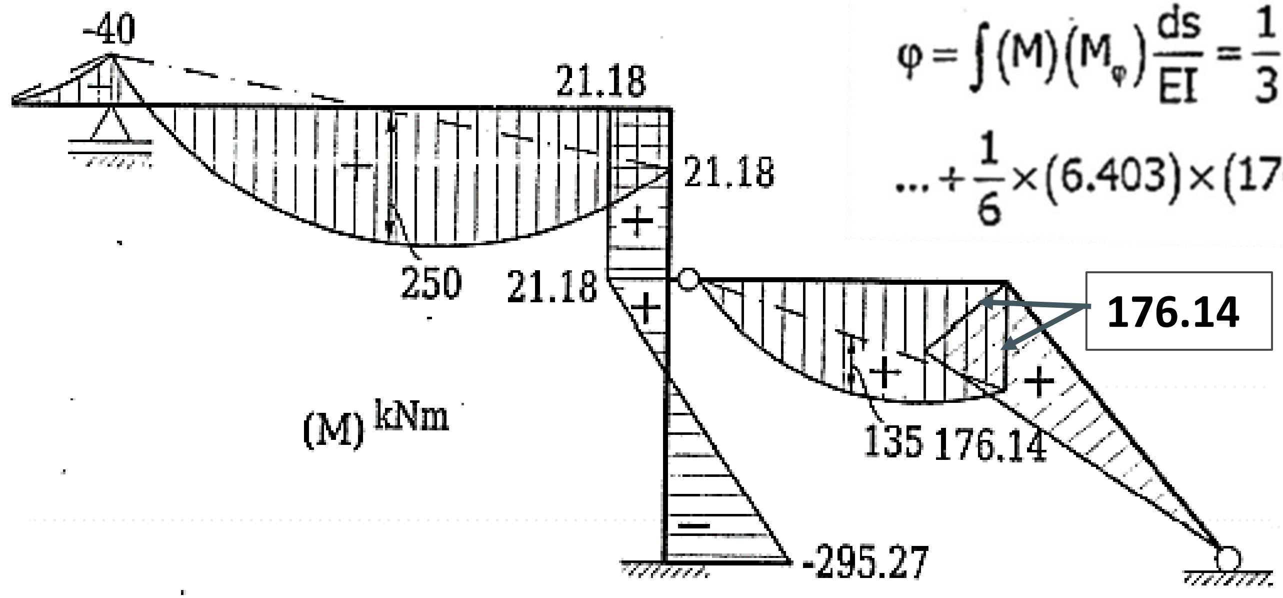
$$\Rightarrow \delta_x = \frac{1186.17}{90000} \cong 0.0132\text{ m}$$

$\varphi = 1$ kNm lik birim yüklemeden oluşan eğilme momenti diyagramı





	k L k	k L	k_1 L k_2	2.° k_m L
i L i	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik$_m$
L i	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li$(2k_1+k_2)$	$\frac{1}{3}$Lik$_m$

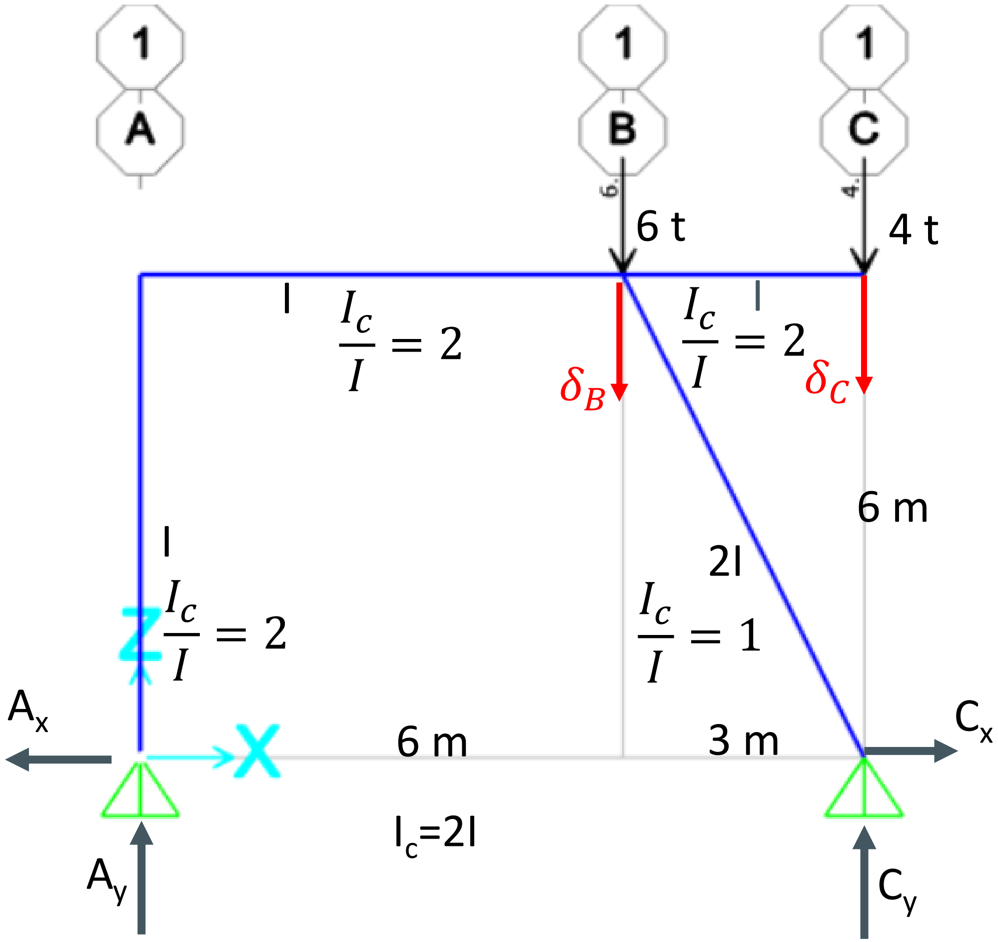


$$\varphi = \int (M)(M_\varphi)\frac{ds}{EI} = \frac{1}{3}\times 6\times(0.60)\times(176.14)\times\frac{1}{3EI} + \frac{1}{3}\times 6\times(0.60)\times(135)\times\frac{1}{3EI} + \ldots$$

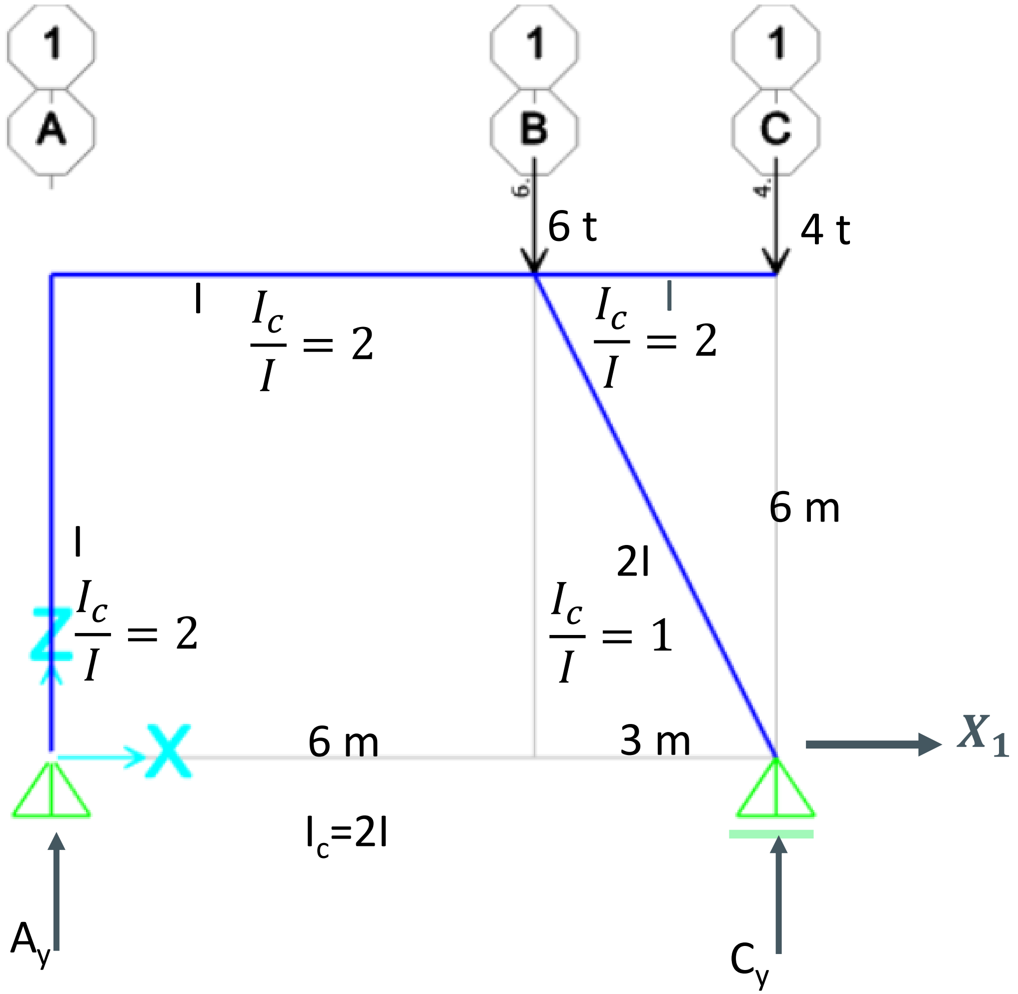
$$\ldots + \frac{1}{6}\times(6.403)\times(176.14)\times(2\times(0.60)+1)\times\frac{1}{2EI} = \frac{331.22}{EI}$$

$$\Rightarrow \varphi = \frac{331.22}{90000} \cong 0.00368 \text{ radyan}$$

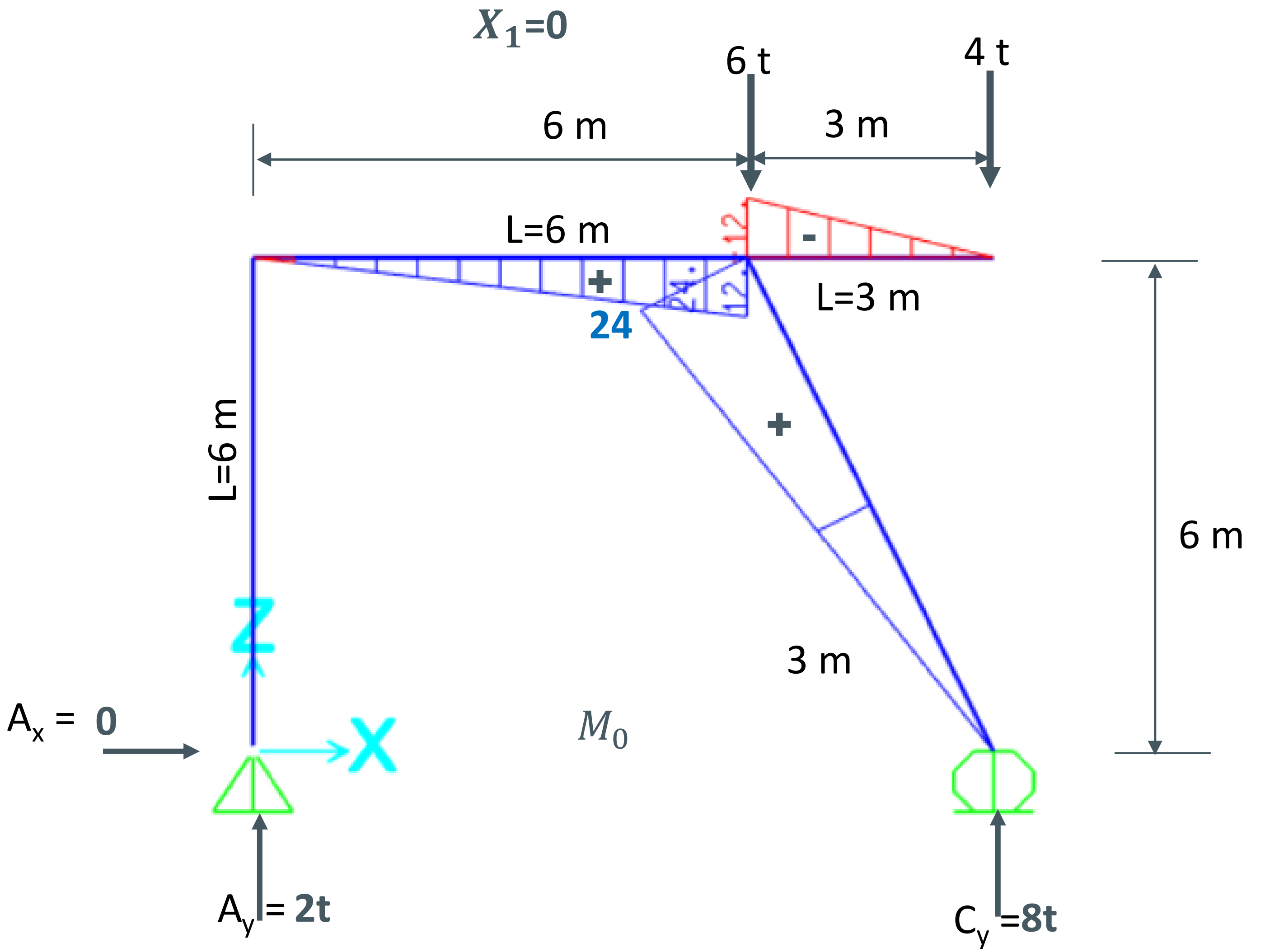
Örnek 11 (Dış yükler altında çözümü)
MNT diyagramlarını çiziniz
δ_B ve δ_C düşey deplasmanlarını bulunuz.

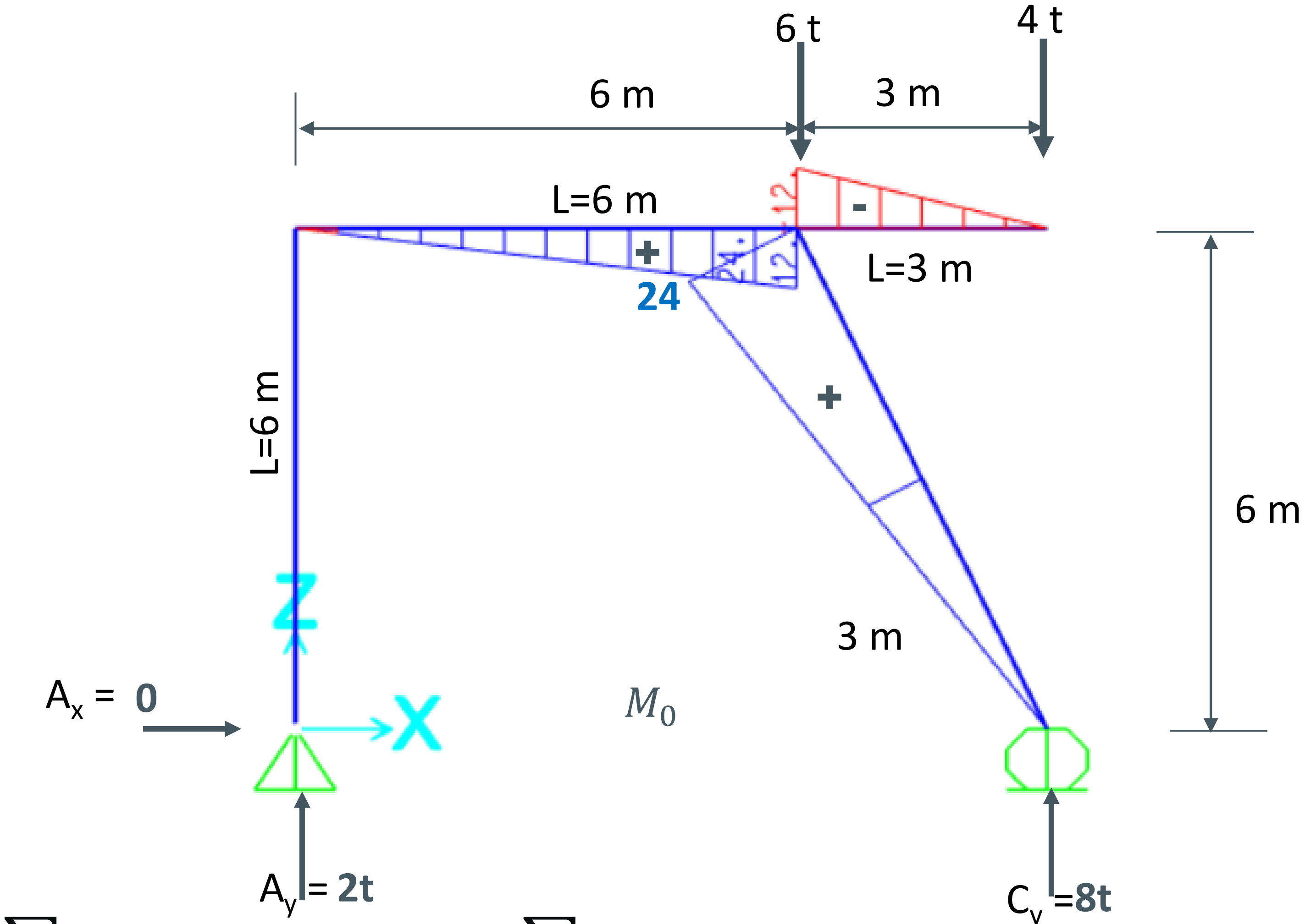


İzostatik Esas Sistem



İzostatik esas sistem(Dış yükler altında çözümü)

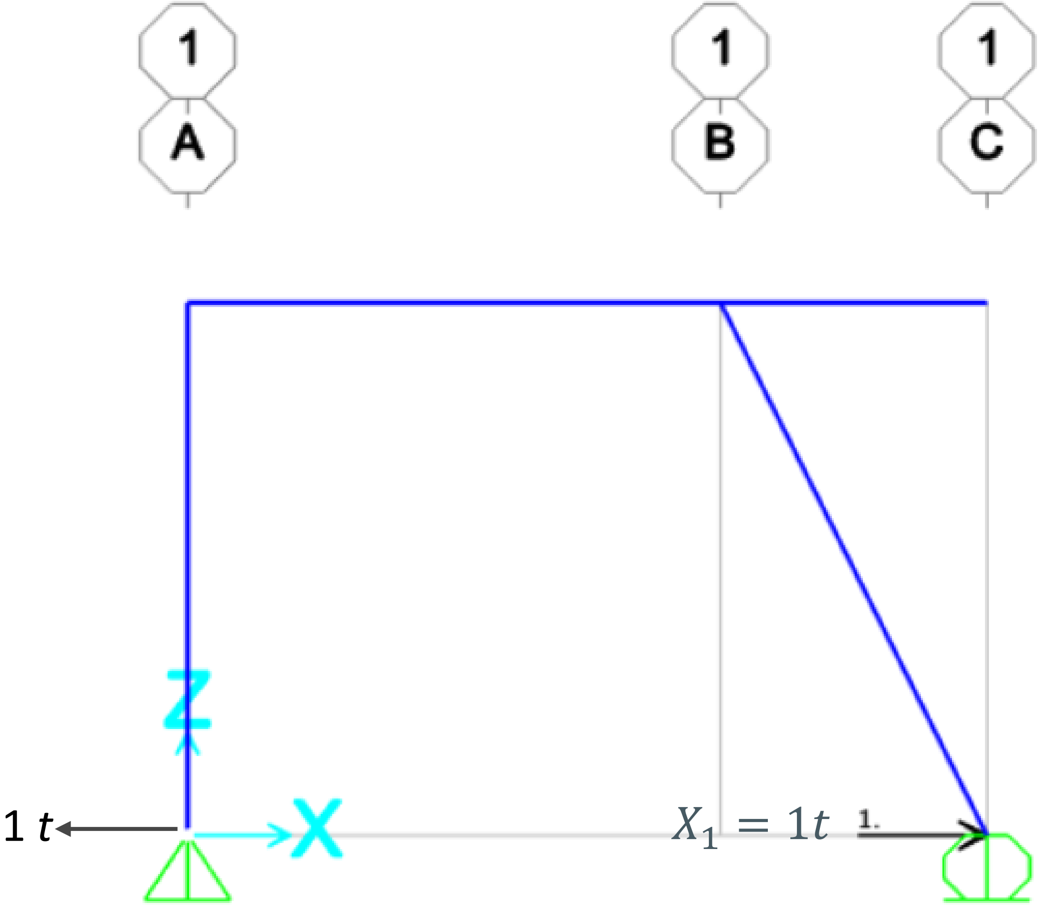


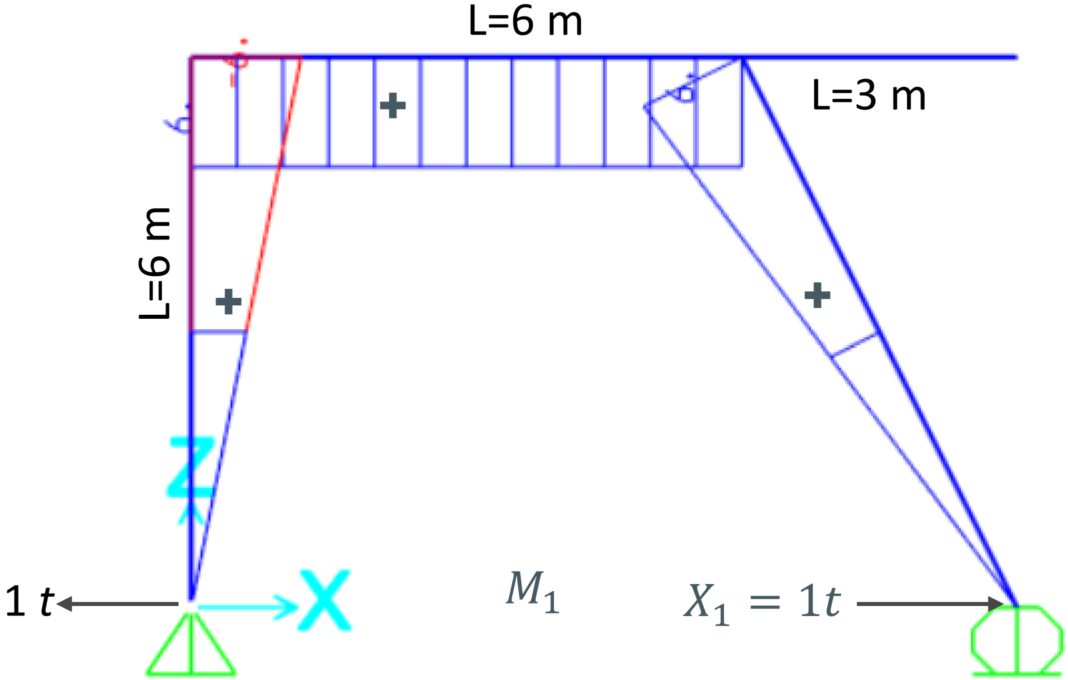


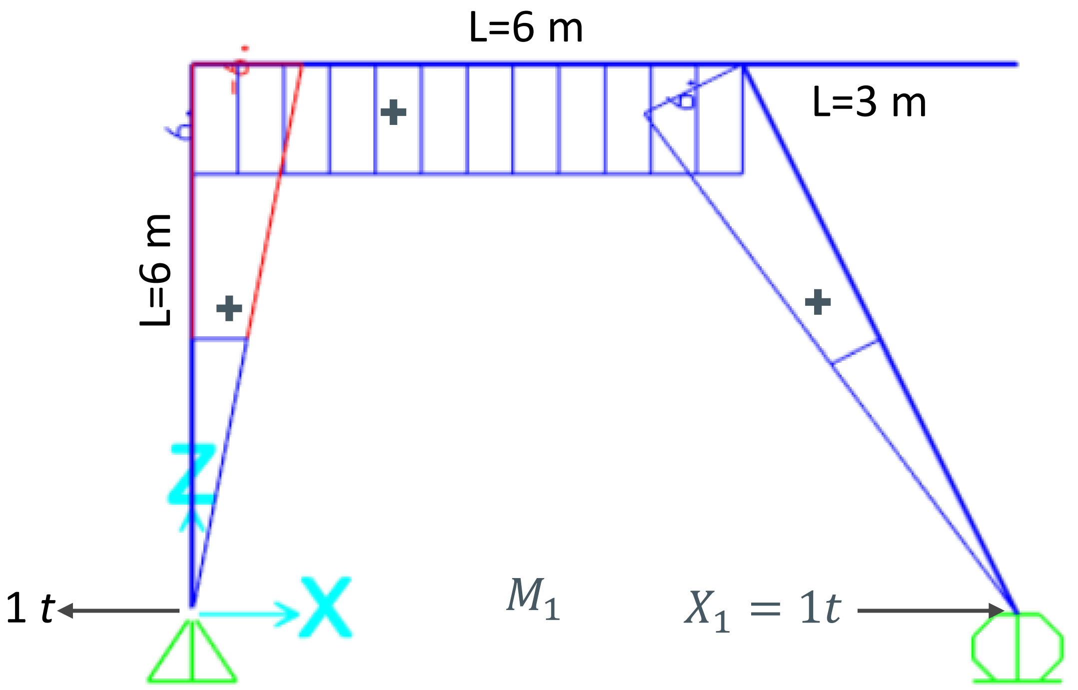
$$\rightarrow^{+} \sum F_x = 0 \;\rightarrow A_x = 0 \qquad \uparrow^{+} \sum F_y = 0 \;\rightarrow A_y + C_y = 10 \rightarrow C_y = 10 - 8 = 2\ t$$

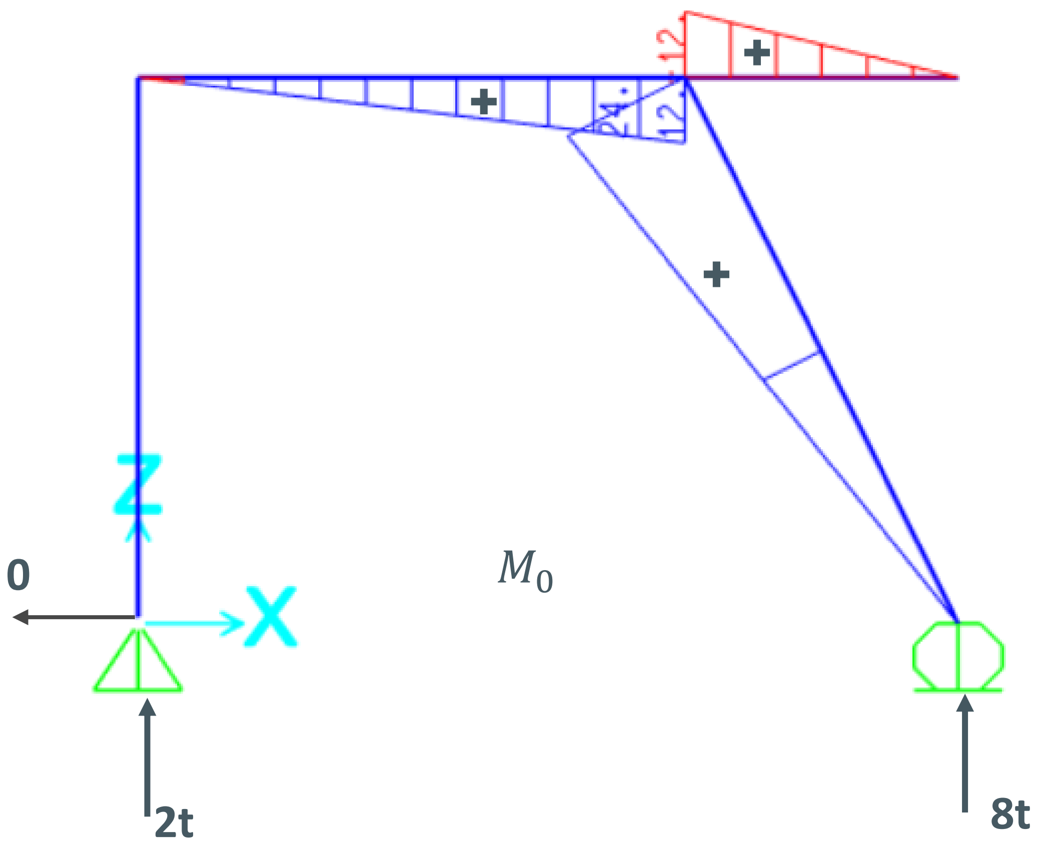
$$\overset{+}{\circlearrowleft} \sum M_A = 0 \;\rightarrow C_y * 9 - 4 * 9 - 6 * 6 = 0 \;\rightarrow C_y = 8\ t$$

$X_1 = 1$ birim yüklemesi



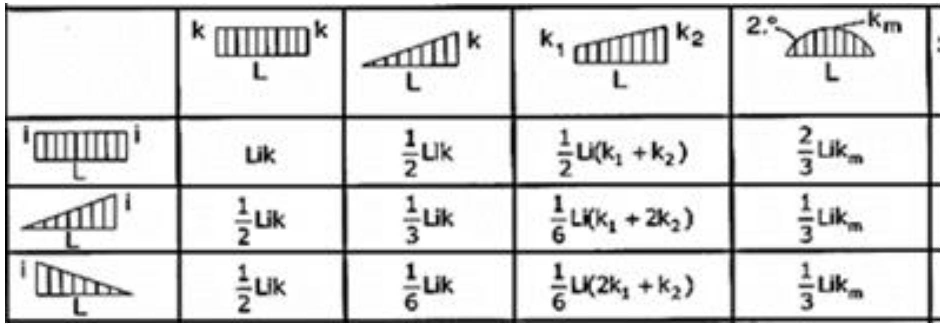






$$EI_c\delta_{11} = \frac{1}{3}6 * 6 * 6 * [2] + 6 * 6 * 6 * [2] + \frac{1}{3}6.708 * 6 * 6 * [1] = 656.496$$

$$EI_c\delta_{10} = \frac{1}{2}6 * 6 * 12 * [2] + \frac{1}{3}6.708 * 6 * 24 * [1] = 753.94$$



	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1 + k_2)	$\frac{2}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1 + 2k_2)	$\frac{1}{3}$Lik_m
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k_1 + k_2)	$\frac{1}{3}$Lik_m

$$EI_c\delta_{11}X_1 + EI_c\delta_{10} = 0$$

$$656.496X_1 + 753.94 = 0 \quad X_1 = -1.1484$$

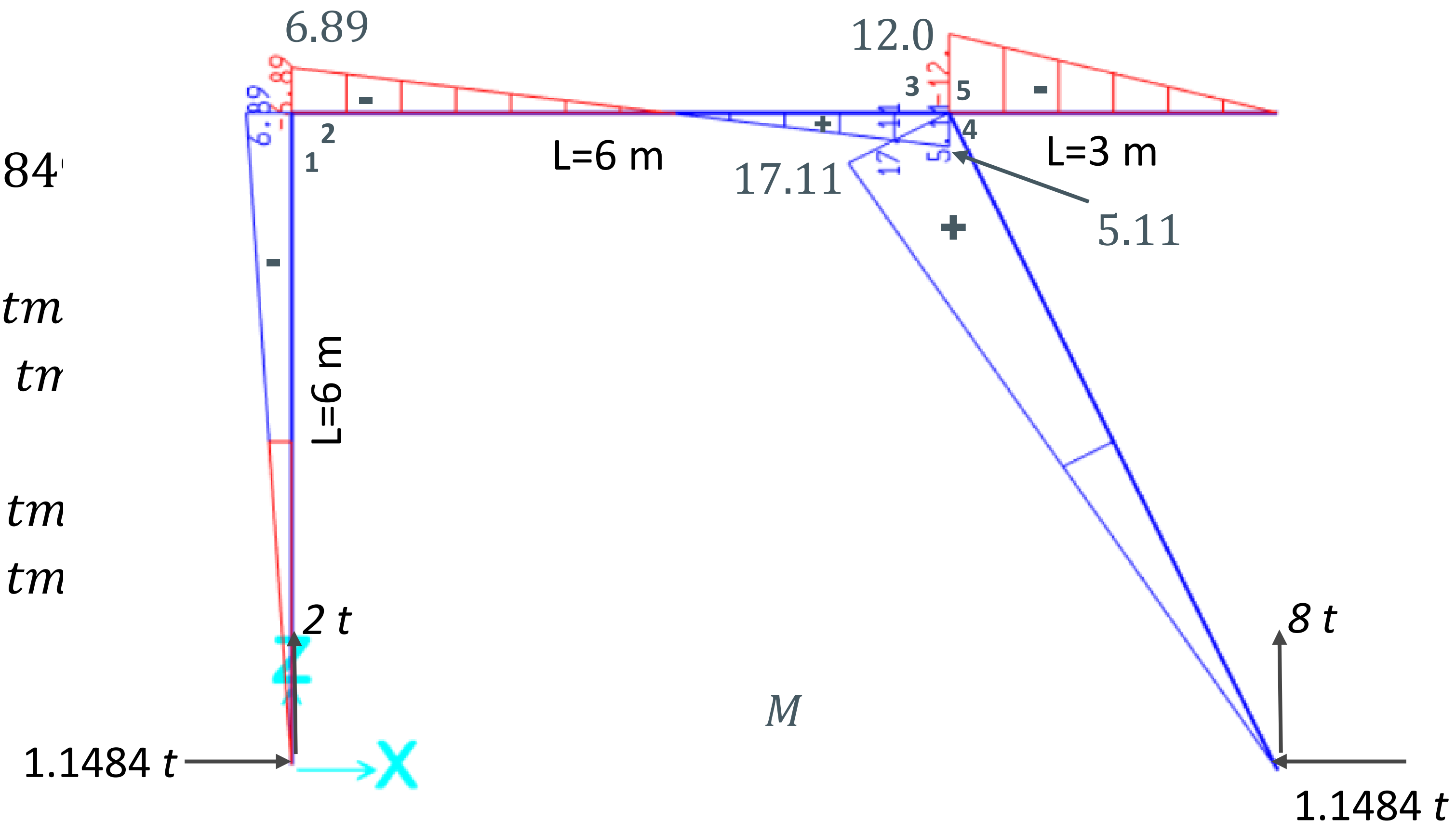
$$M_{(3)} = 12 + 6(-1.1485) = 5.109\ tm$$

$$M_{(4)} = 24 + 6(-1.1485) = 17.109\ tn$$

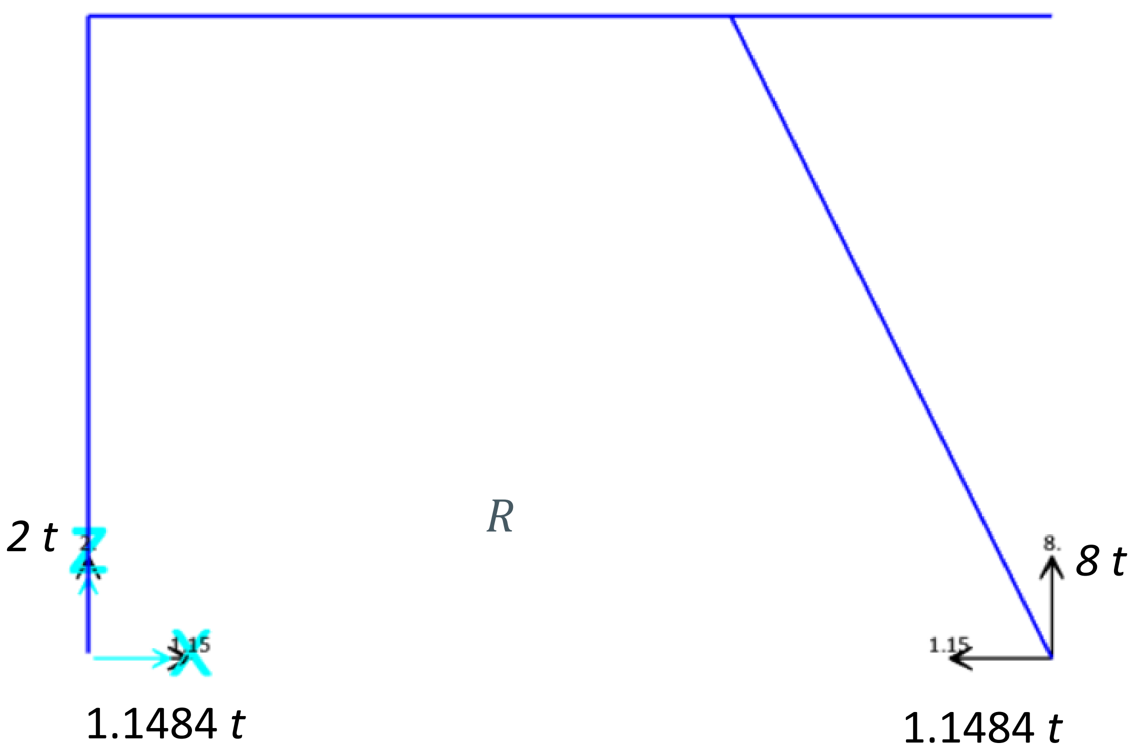
$$M_{(5)} = -12 + 0 = -12\ tm$$

$$M_{(1)} = 0 + 6(-1.1485) = -6.891\ tm$$

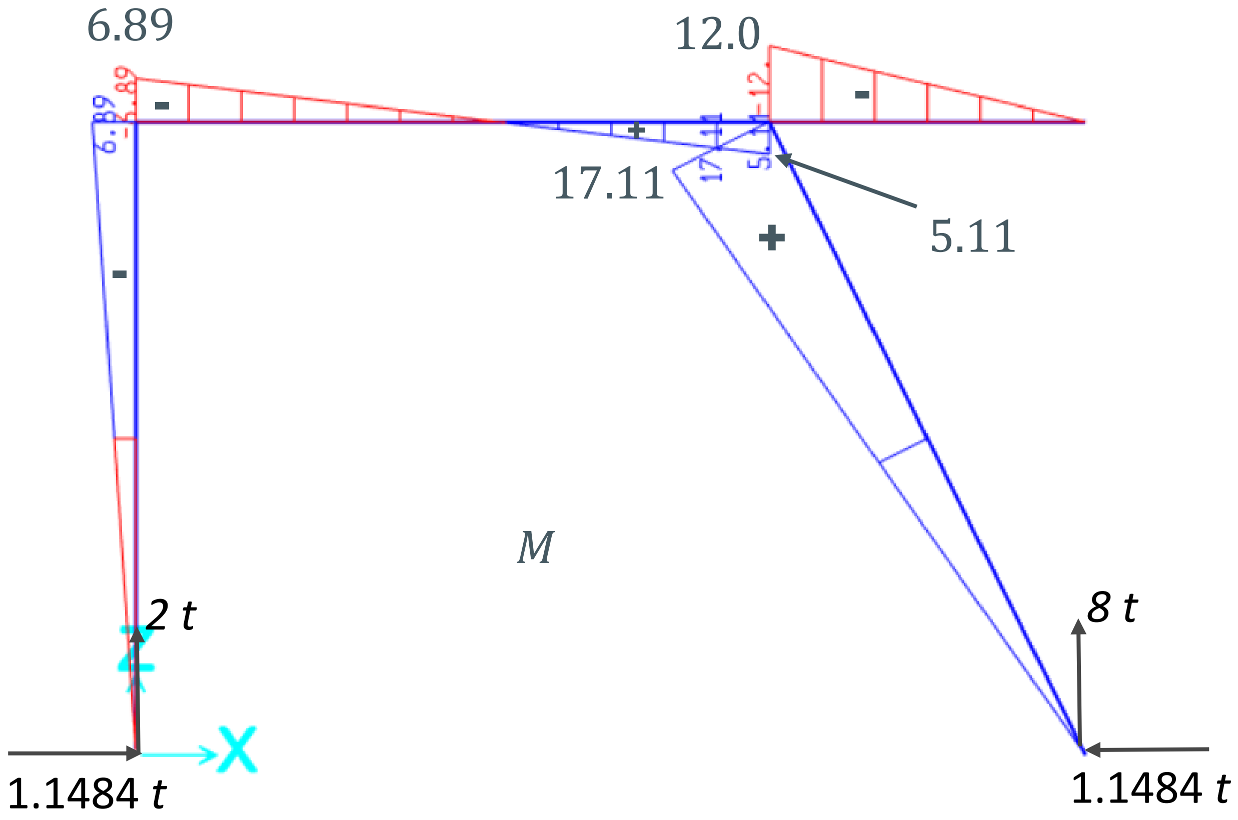
$$M_{(2)} = 0 + 6(-1.1485) = -6.891\ tm$$



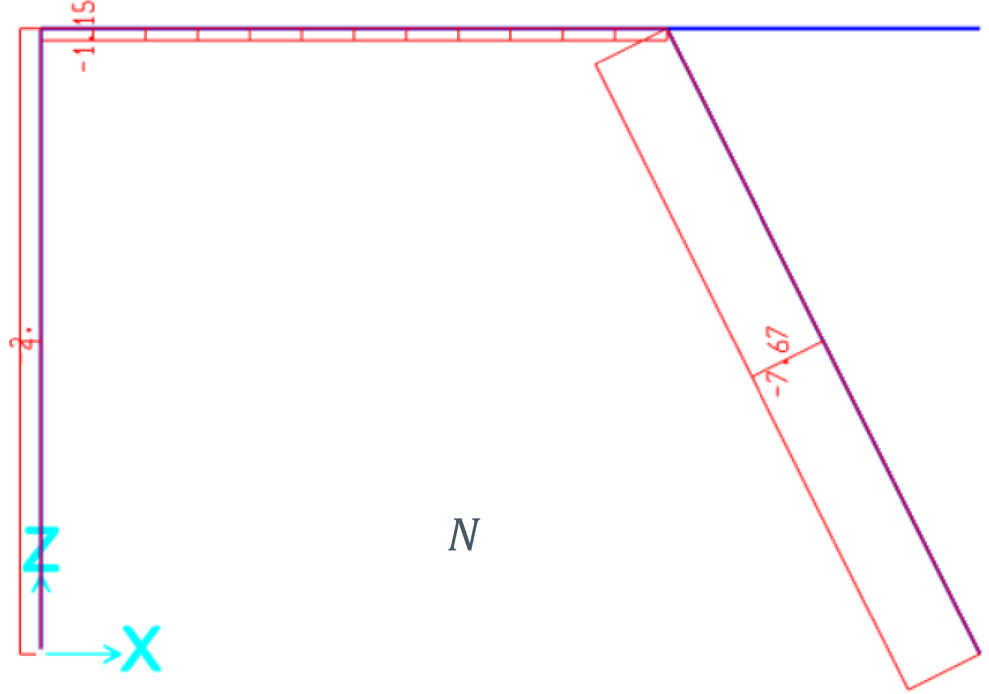
Mesnet tepkileri



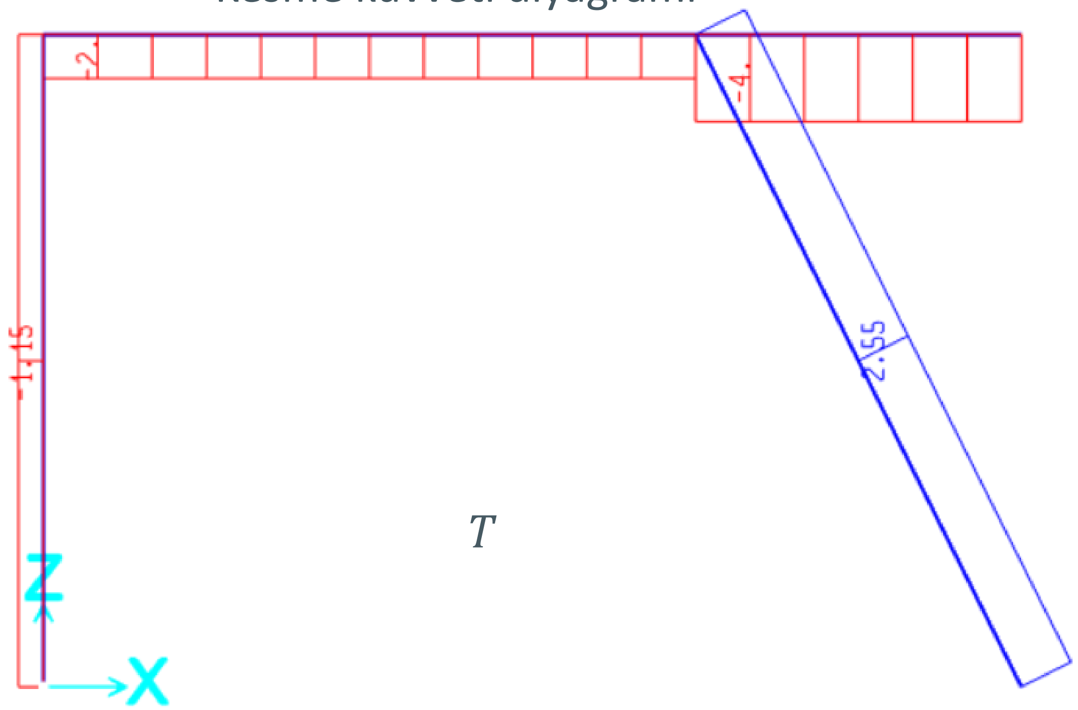
Moment diyagramı



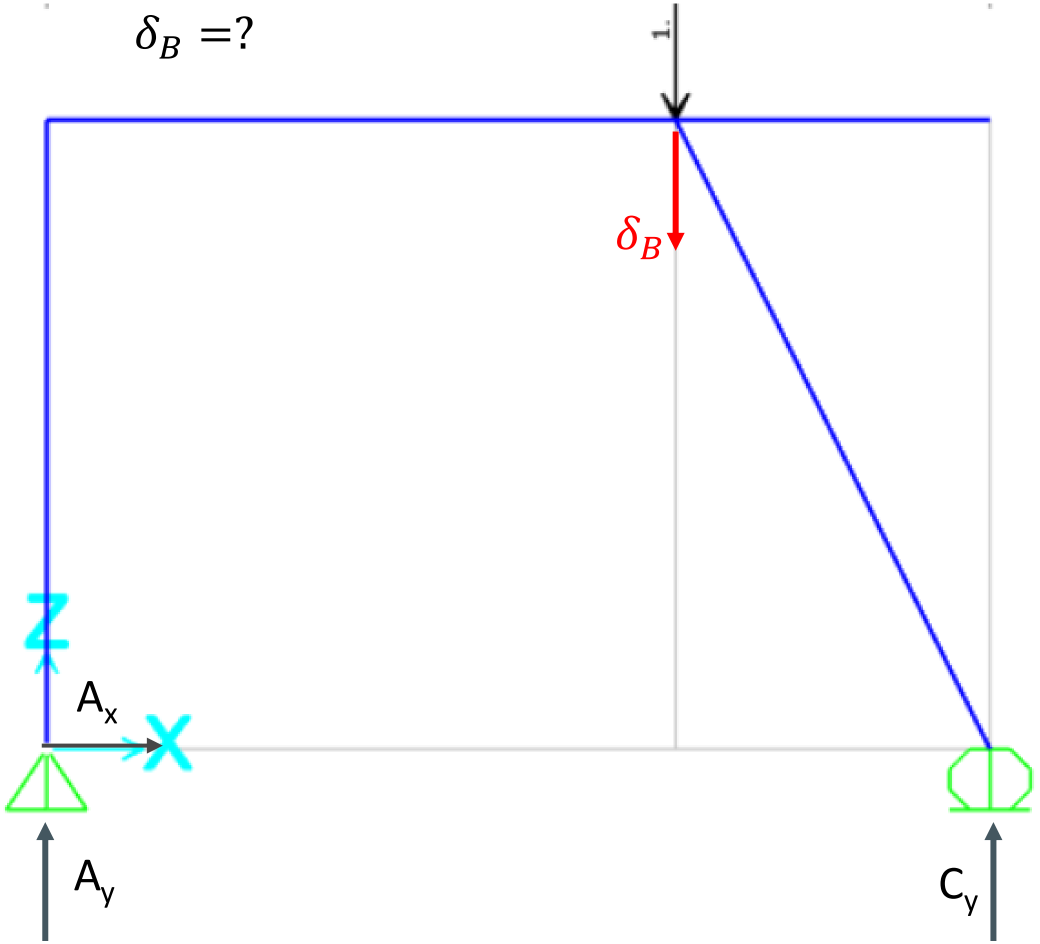
Normal kuvvet diyagramı



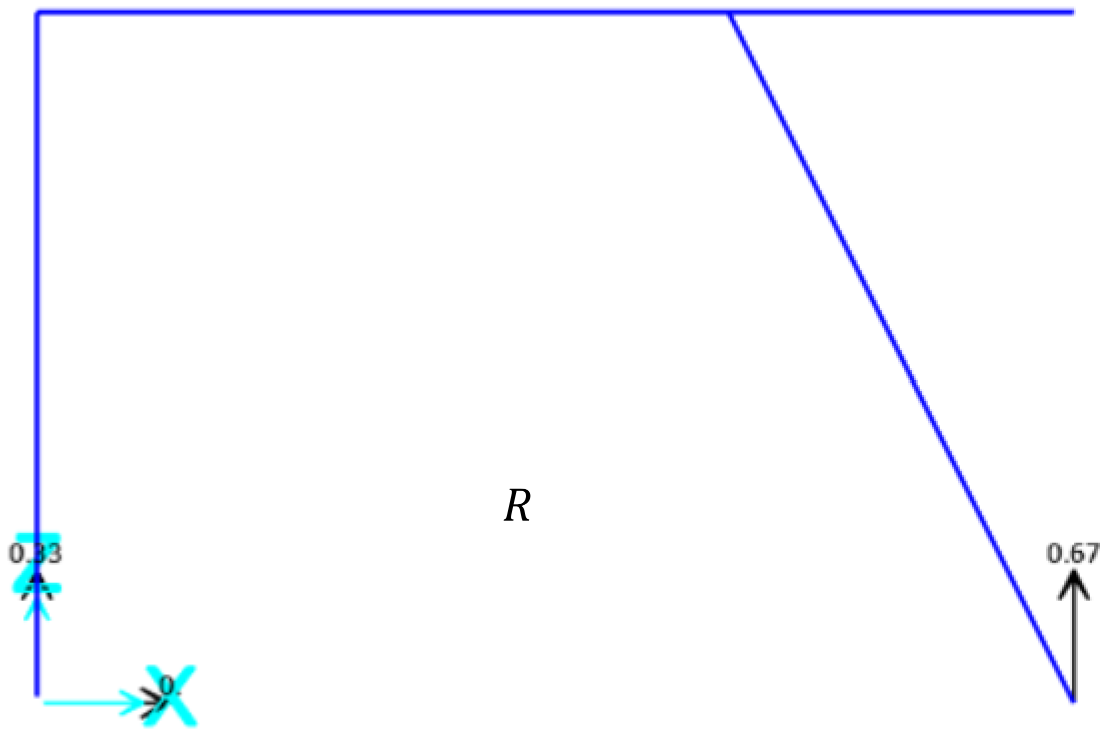
Kesme kuvveti diyagramı



Deplasman yönünde birim yükleme



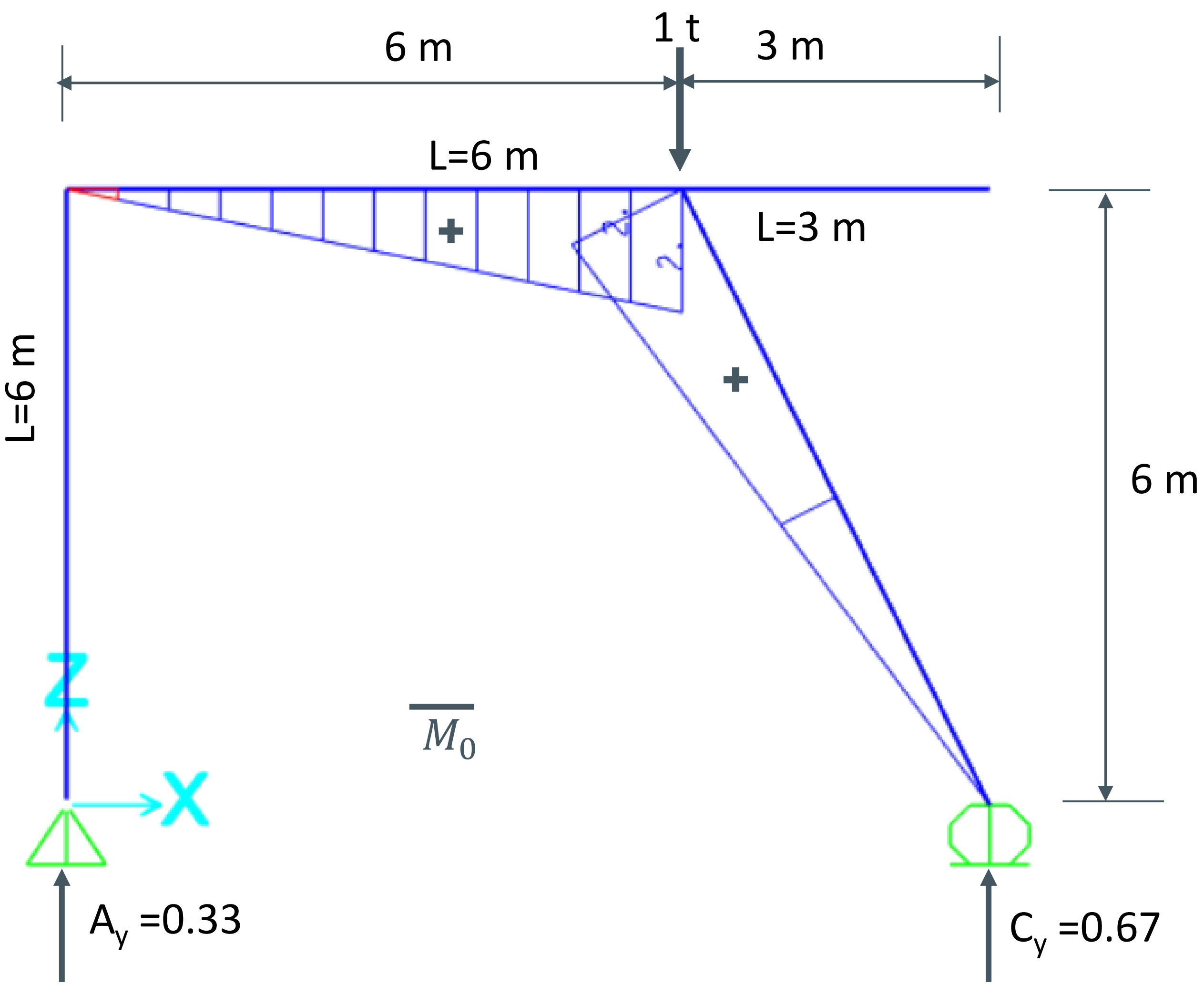
Mesnet tepkileri



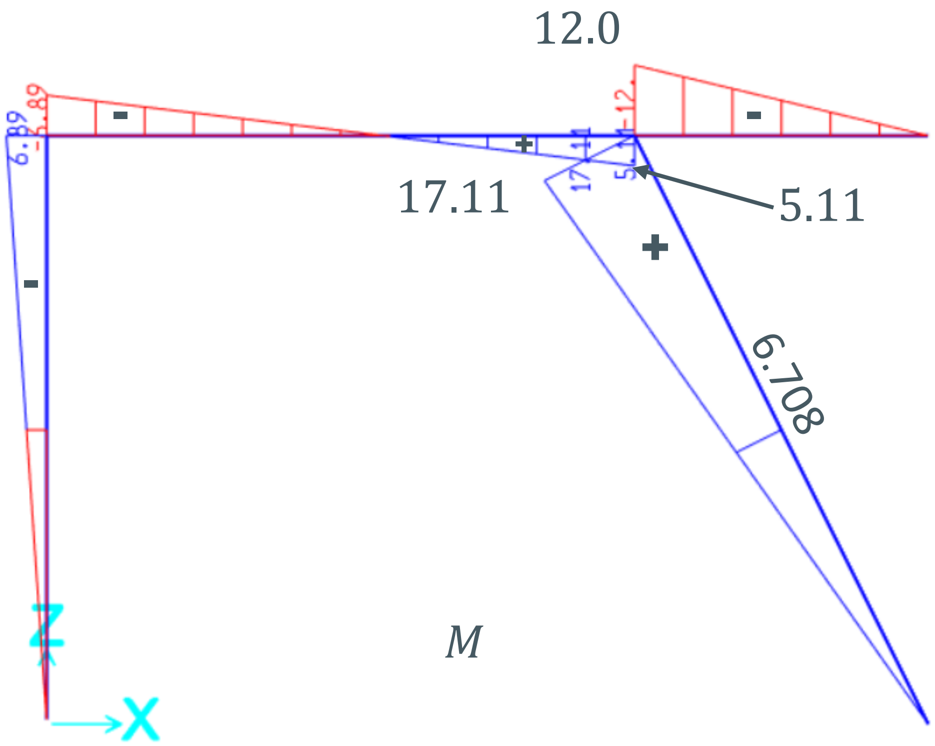
$$\circlearrowleft^{+} \sum M_A = 0 \;\rightarrow C_y * 9 - 1 * 6 = 0 \;\rightarrow C_y = \frac{6}{9} = 0.67\ t$$

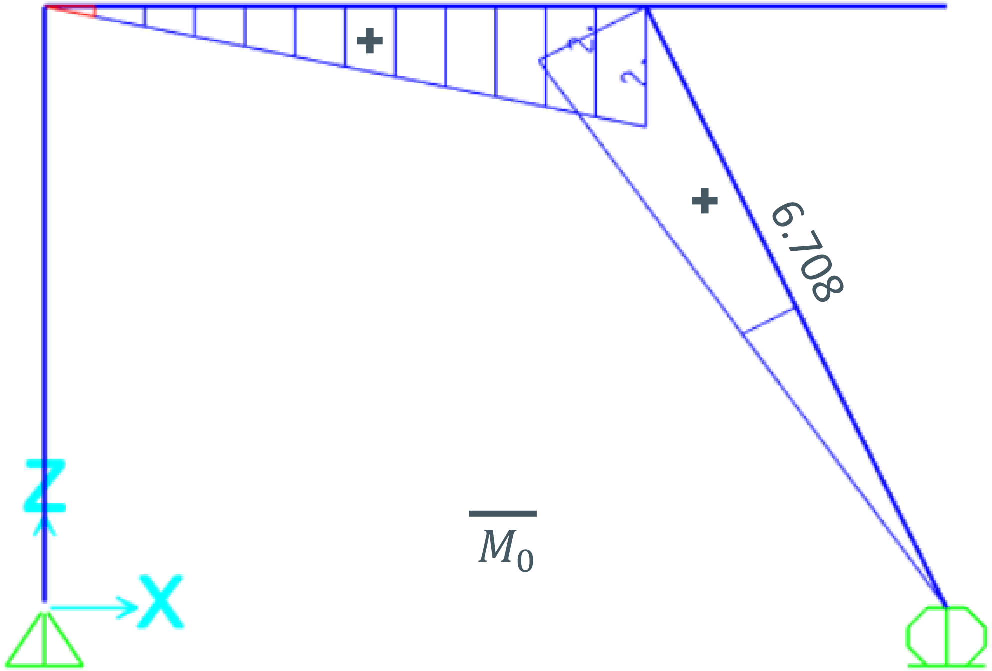
$$\rightarrow^{+} \sum F_x = 0 \;\rightarrow A_x = 0 \qquad \uparrow^{+} \sum F_y = 0 \;\rightarrow A_y + C_y = 1 \rightarrow C_y = 1 - 0.67 = 0.33\ t$$

Deplasman yönünde birim yükleme moment diyagramı



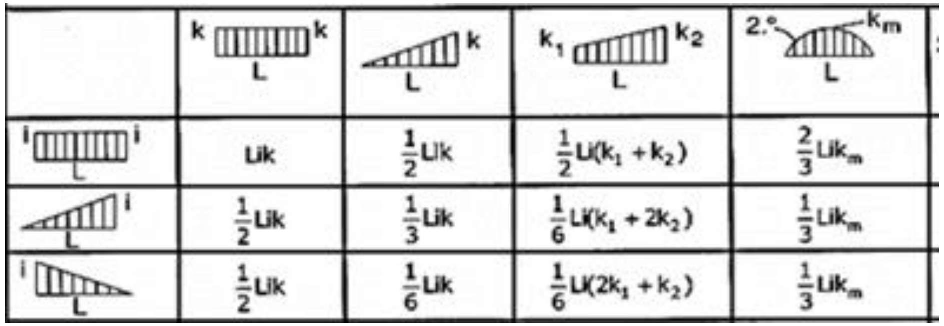
$$EI_c = 2.1 * 10^6 * 80 * 10^{-4} = 16800\ tm^2$$





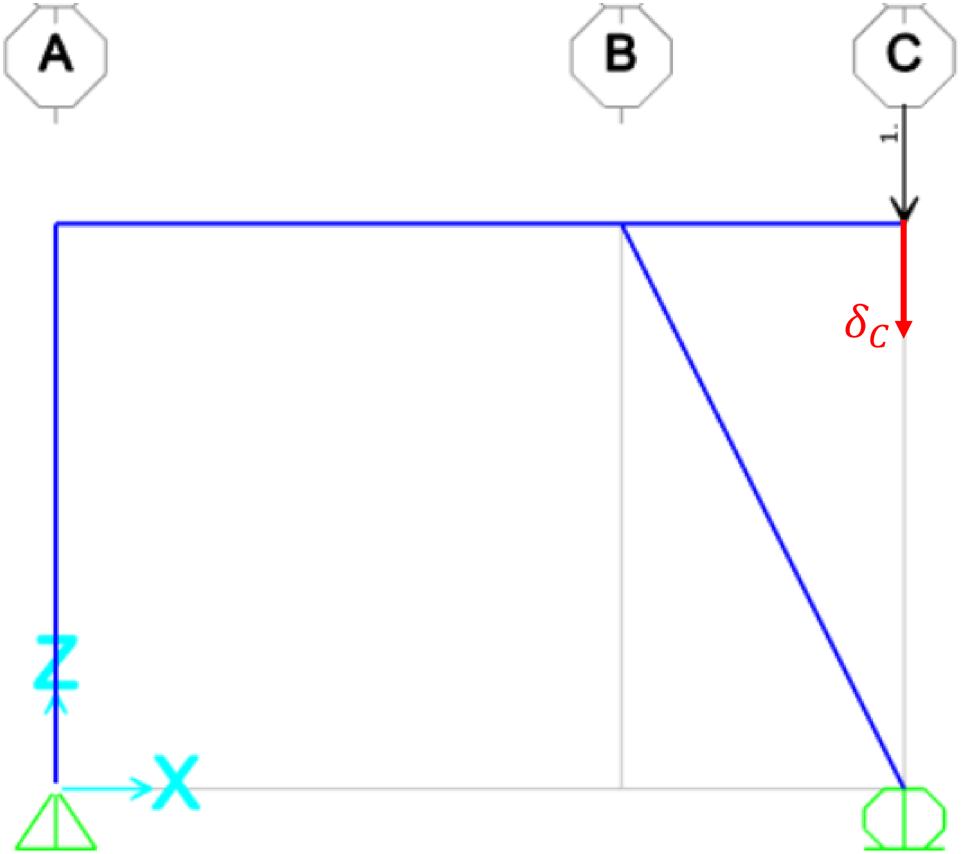
$$EI_c\delta_B = \frac{1}{6}6 * 2 * (-6.89 + 2 * 5.11)[2] + \frac{1}{3}6.708 * 17.11 * 2 * [1] = 89.83$$

$$\delta_B = \frac{89.83}{E(2I)} = \frac{44.92}{EI}m$$

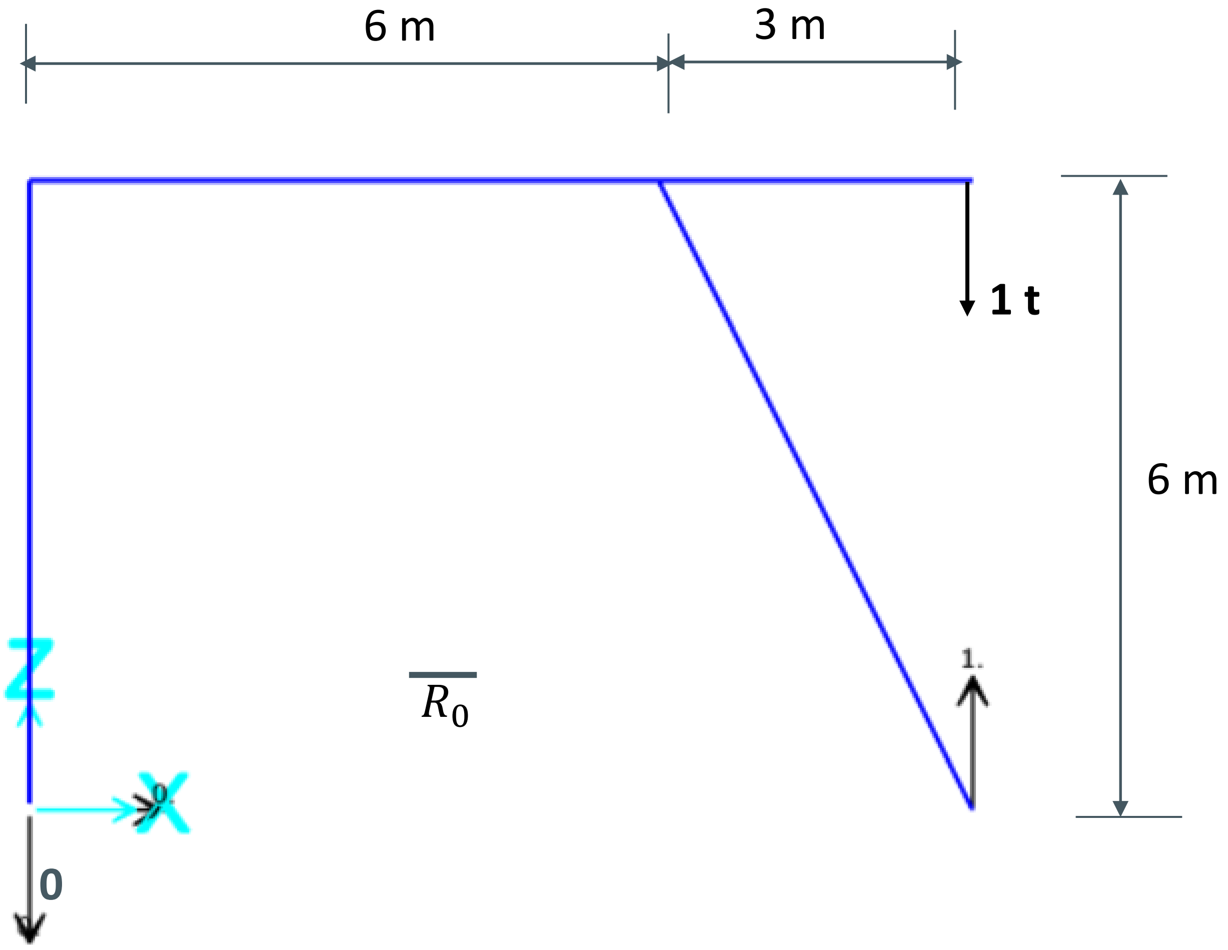


	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1 + k_2)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1 + 2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k_1 + k_2)	$\frac{1}{3}$Lik$_m$

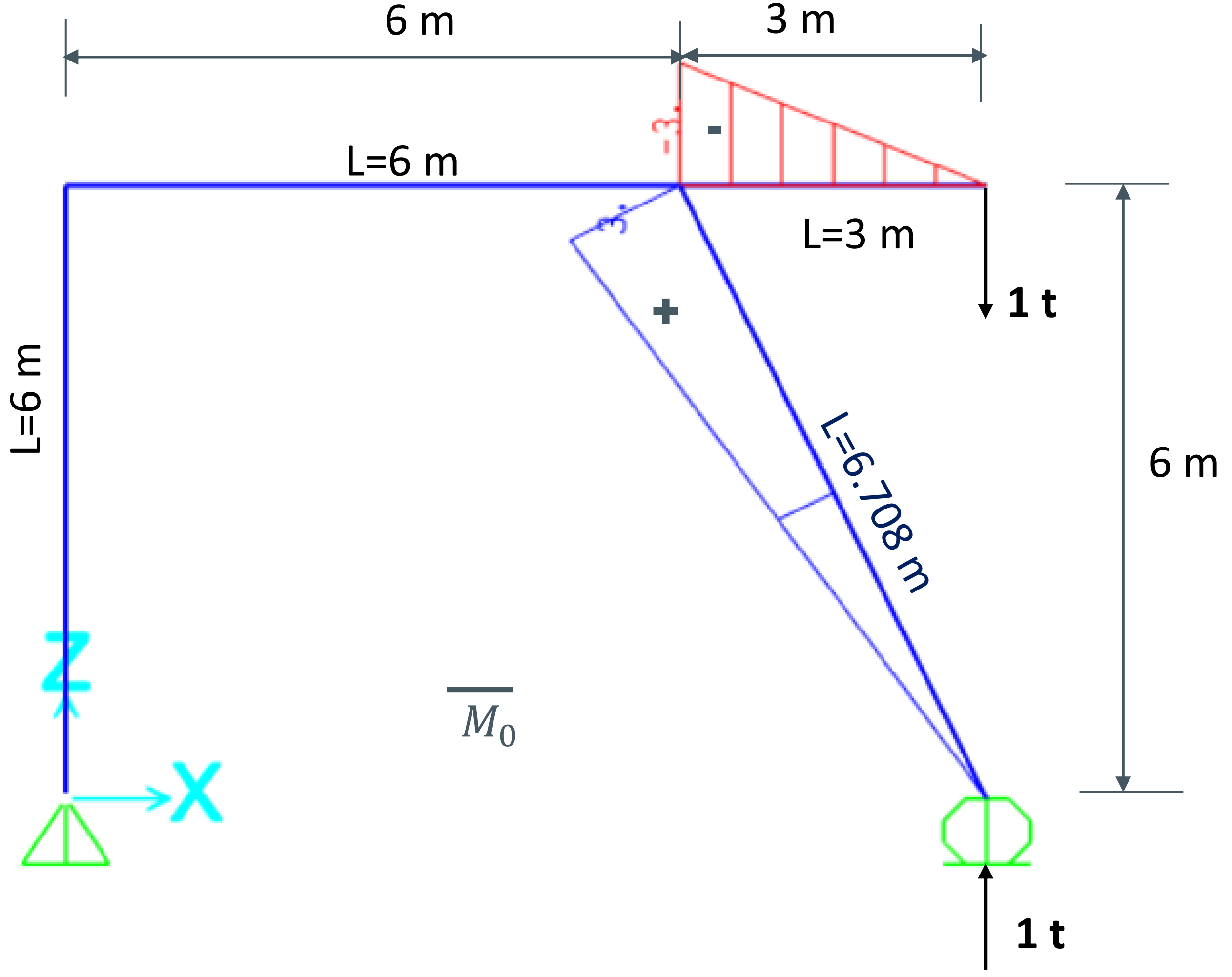
Deplasman yönünde birim yükleme

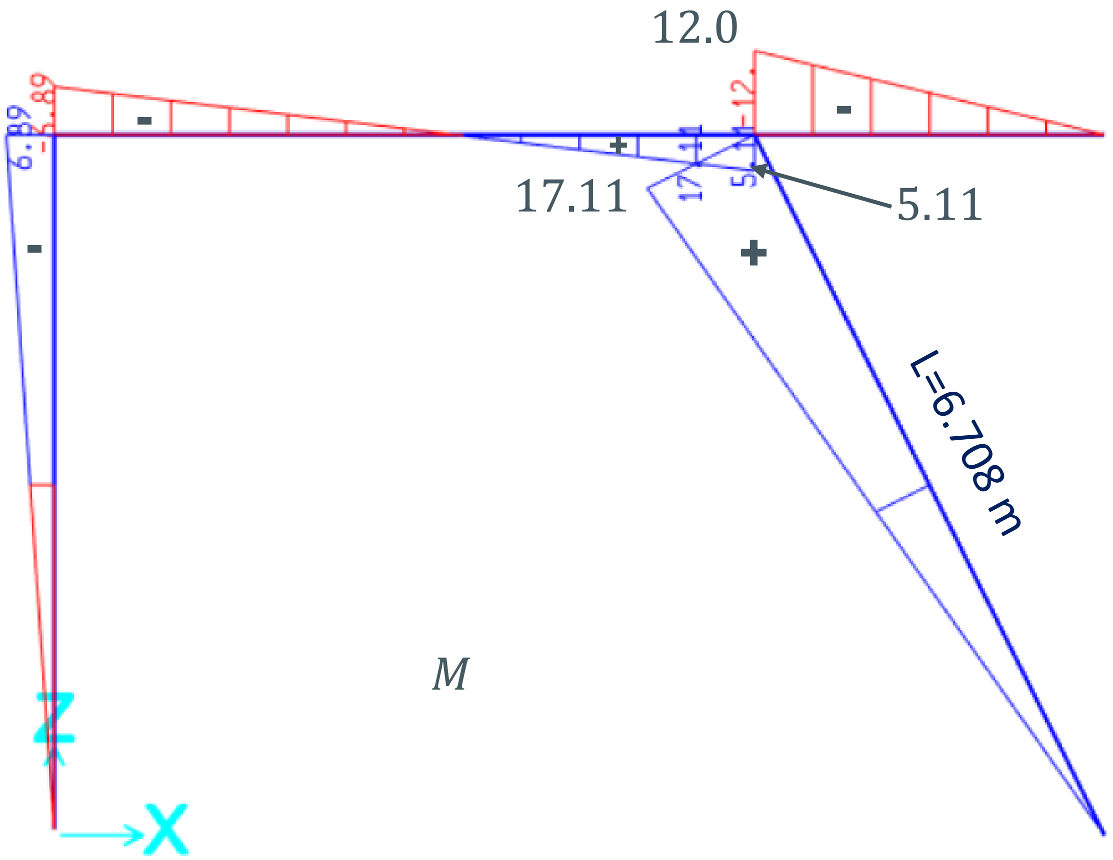


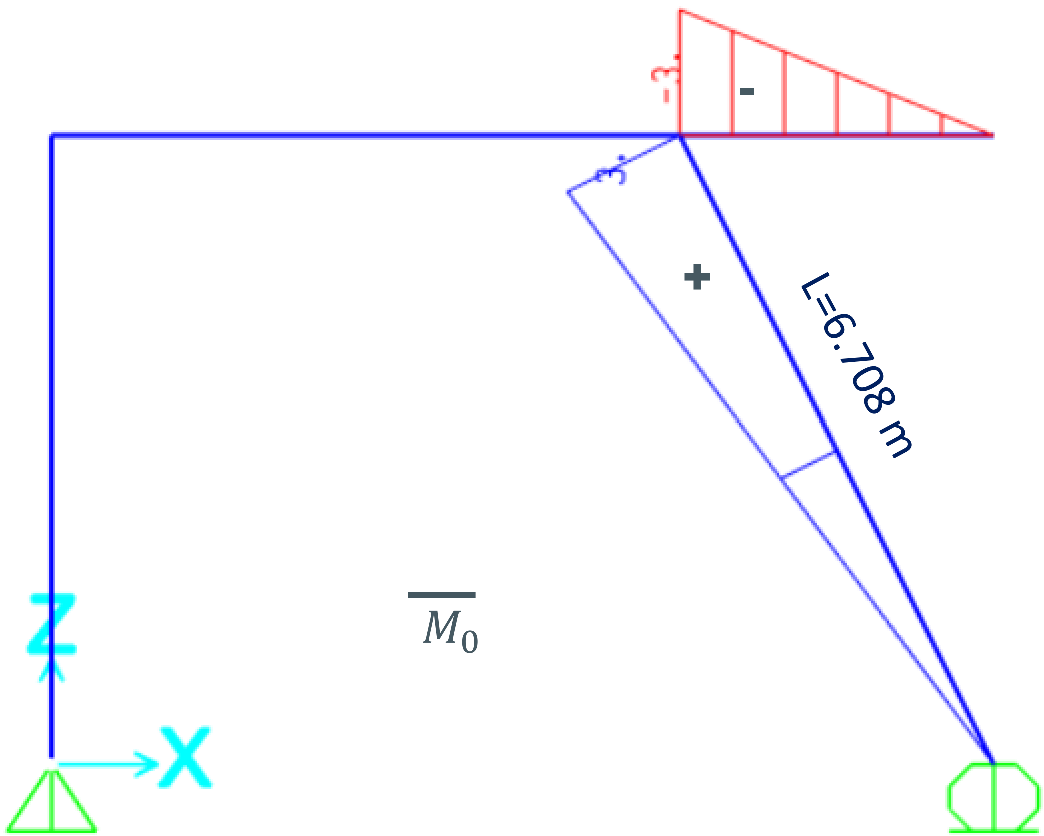
Mesnet tepkileri



Deplasman yönünde birim yükleme moment diyagramı

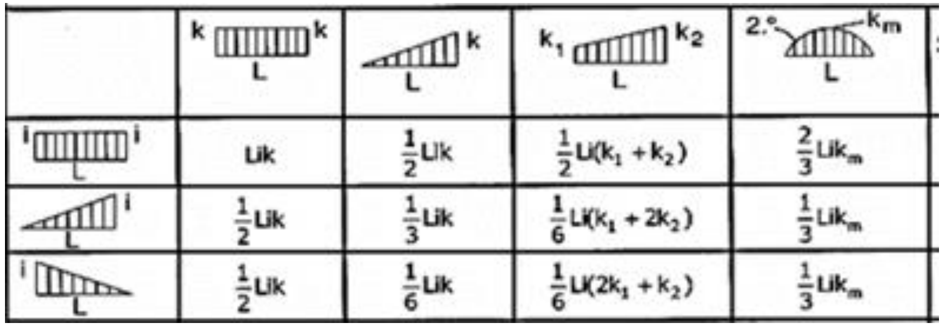






$$EI_c\delta_C = \frac{1}{3}3*(-12)*(-3)[2] + \frac{1}{3}6.708*17.11*(3)*[1] = 186.77$$

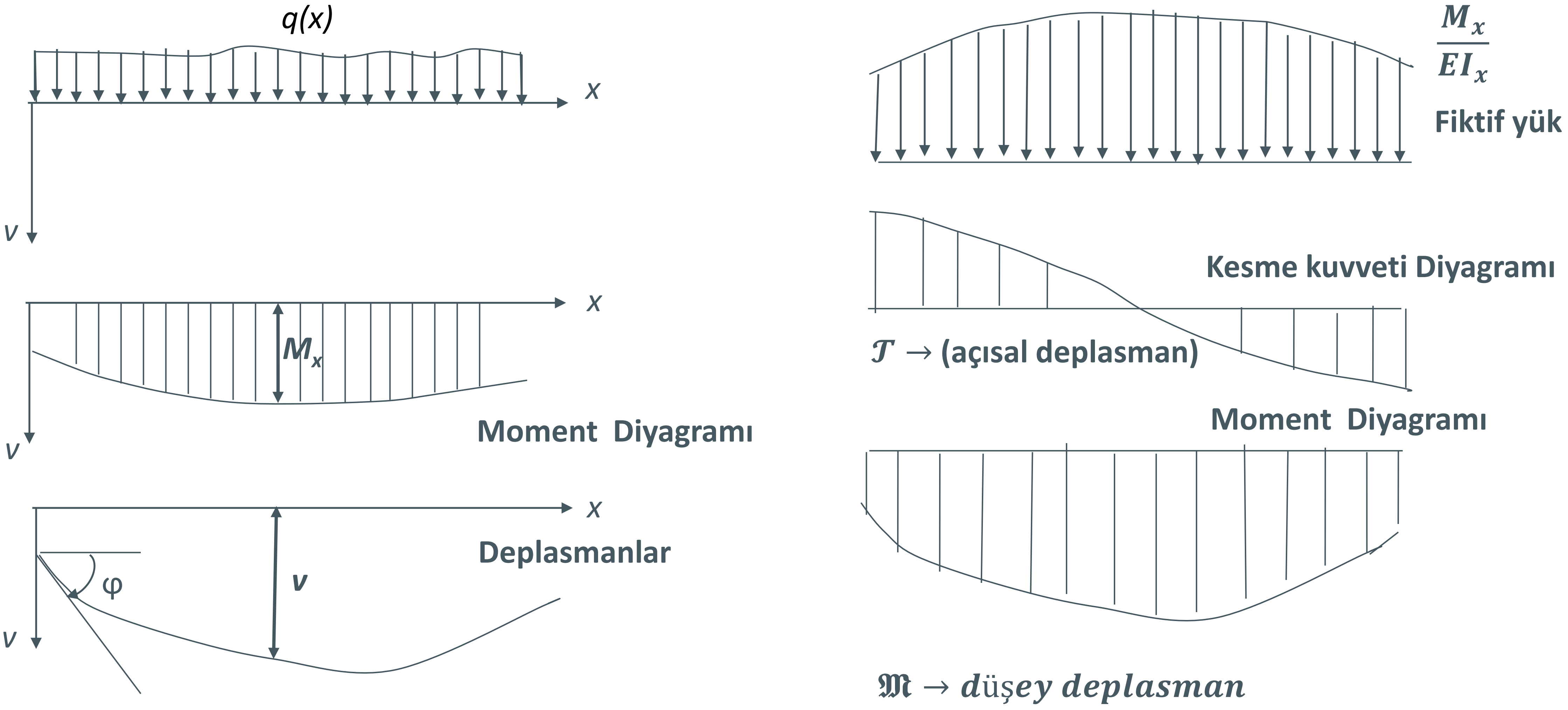
$$\delta_C = \frac{186.77}{E(2I)} = \frac{93.386}{EI}m$$



	k k L	k L	k₁ k₂ L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$L(k₁ + k₂)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k₁ + 2k₂)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k₁ + k₂)	$\frac{1}{3}$Lik$_m$

2. MOHR METODU İLE DEPLASMAN HESABI

Dolu gövdeli sistemlerde deplasman hesabında kullanılır.



Moment Diyagramı

Deplasmanlar

$$\frac{M_x}{EI_x}$$

Fiktif yük

Kesme kuvveti Diyagramı

$\mathcal{T} \rightarrow$ **(açısal deplasman)**

Moment Diyagramı

$\mathfrak{M} \rightarrow \boldsymbol{düşey\ deplasman}$

Deformasyon denklemleri

$$\frac{d^2v}{dx^2} = -\frac{M(x)}{EI(x)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\,.\ldots(1)$$

$$\frac{d^2v(x)}{dx^2} = \frac{d^2\mathfrak{M}}{dx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\,(2)$$

(2) Denkleminin bir kere integrali alınırsa

$$\varphi_x = \frac{dv(x)}{dx} = \frac{d\mathfrak{M}(x)}{dx} + C_1 = \boldsymbol{\mathcal{T}}(x) + C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\,.\,(3)$$

(2) Denkleminin tekrar integrali alınırsa

$$v(x) = \mathfrak{M}(x) + C_1x + C_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots\ldots.\,(4)$$

3 ve 4 denklemlerinden bir sistemin dönmeleri bir sabit farkı ile fiktif sistemin dönmelerine çökmeleri de eğilme momentlerine eşittir. Ancak bu analojinin uygulanabilmesi için sınır şartları da birbirine benzer olması gerekir. Yani gerçek sistemlerde sınırlardaki $\boldsymbol{v}$ ve $\boldsymbol{\varphi}$ ler fiktif sistemde $\mathfrak{M}$ ve $\boldsymbol{\mathcal{T}}$ lara karşı gelmektedir.

Fiktif Sistemin Özellikleri :

1. Fiktif sistem gerçek sistemde geometrik sınır şartları bilinen iki nokta arasında alınan doğru eksenli yatay bir çubuktur.
2. Fiktif sistemin mesnetleri gerçek sistemde geometrik sınır şartları bilinen iki nokta arasında $\boldsymbol{v}$ ler $\mathfrak{M}$ ye $\boldsymbol{\varphi}$ ler de $\boldsymbol{\mathcal{T}}$ ya karşılık gelecek şekilde tayin edilir.
3. İzostatik fiktif sistem izostatik olmalıdır.

Fiktif Sistemlerin Seçimi :

$$M_A = 0$$

$$v_A = 0 \quad \mathfrak{M}_A = 0 \quad v_B = 0 \quad \mathfrak{M}_B = 0$$

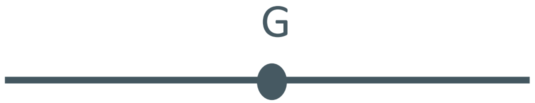
$$\varphi_A \neq 0 \quad \boldsymbol{\mathcal{T}}_A \neq 0 \quad \varphi_B \neq 0 \quad \boldsymbol{\mathcal{T}}_B \neq 0$$

Gerçek

Fiktif

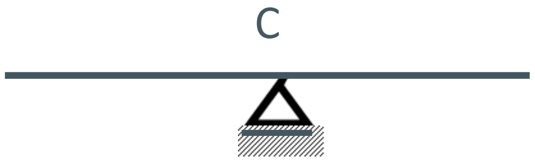
$v_A = 0 \quad \mathfrak{M}_A = 0 \quad v_B \neq 0 \quad \mathfrak{M}_B \neq 0$

$\varphi_A = 0 \quad \mathcal{T}_A = 0 \quad \varphi_B \neq 0 \quad \mathcal{T}_B =\neq 0$



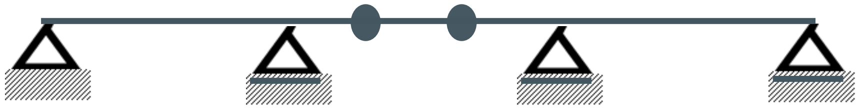
$v_G \neq 0 \quad \mathfrak{M}_G \neq 0$

$\varphi_G \neq 0 \quad \mathcal{T}_G \neq 0$



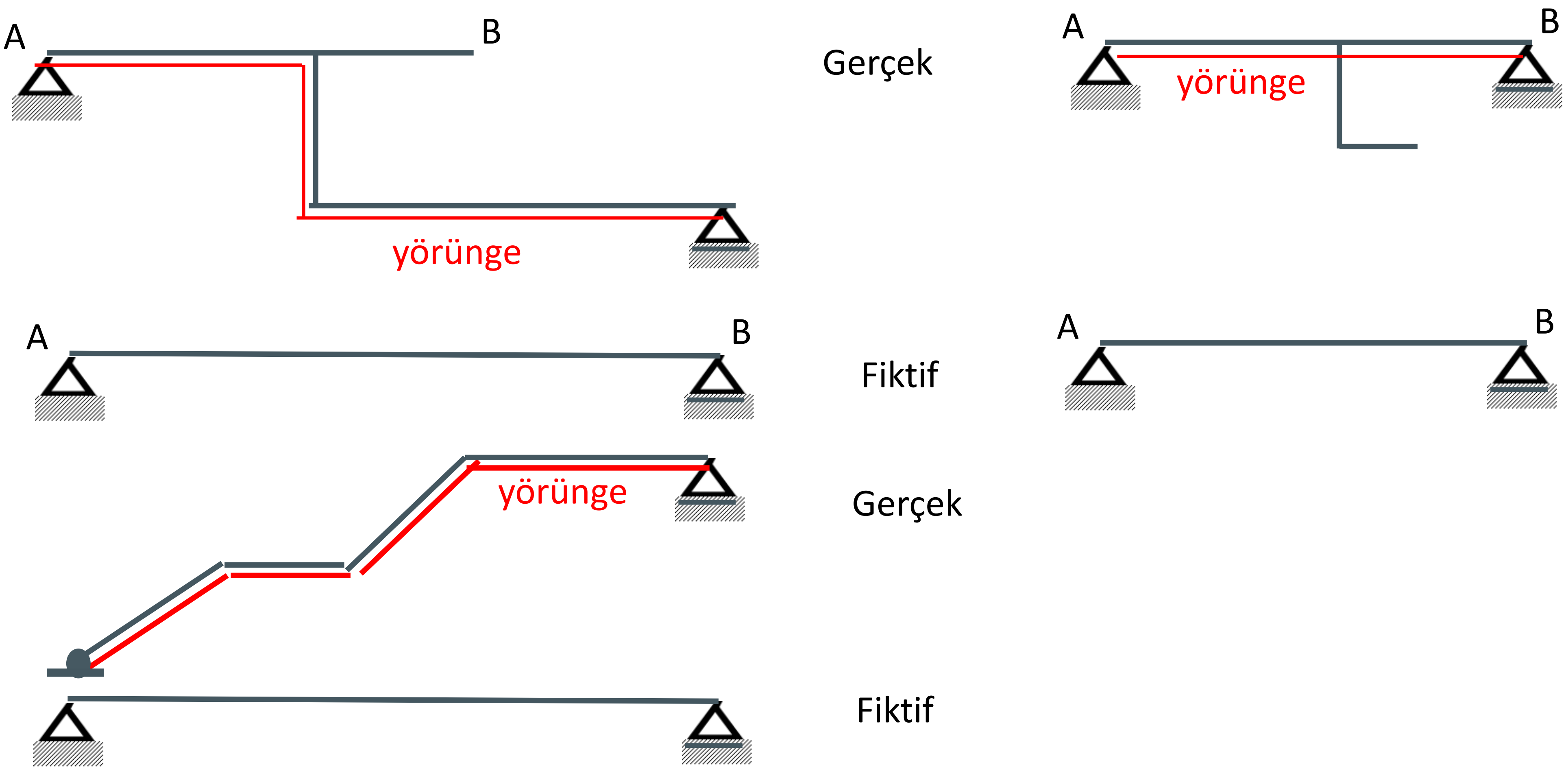
$v_C = 0 \quad \mathfrak{M}_C = 0$

$\varphi_C \neq 0 \quad \mathcal{T}_C \neq 0$



Fiktif Sistemin Seçimi için Yardımcı Bilgiler :

1. Gerçek sistemde geometrik sınır şartları bilinen iki nokta arasındaki yola yörünge denir.



2. Mohr metodu ile yalnız yörünge üzerindeki noktaların deplasmanı direkt olarak tayin edilir.

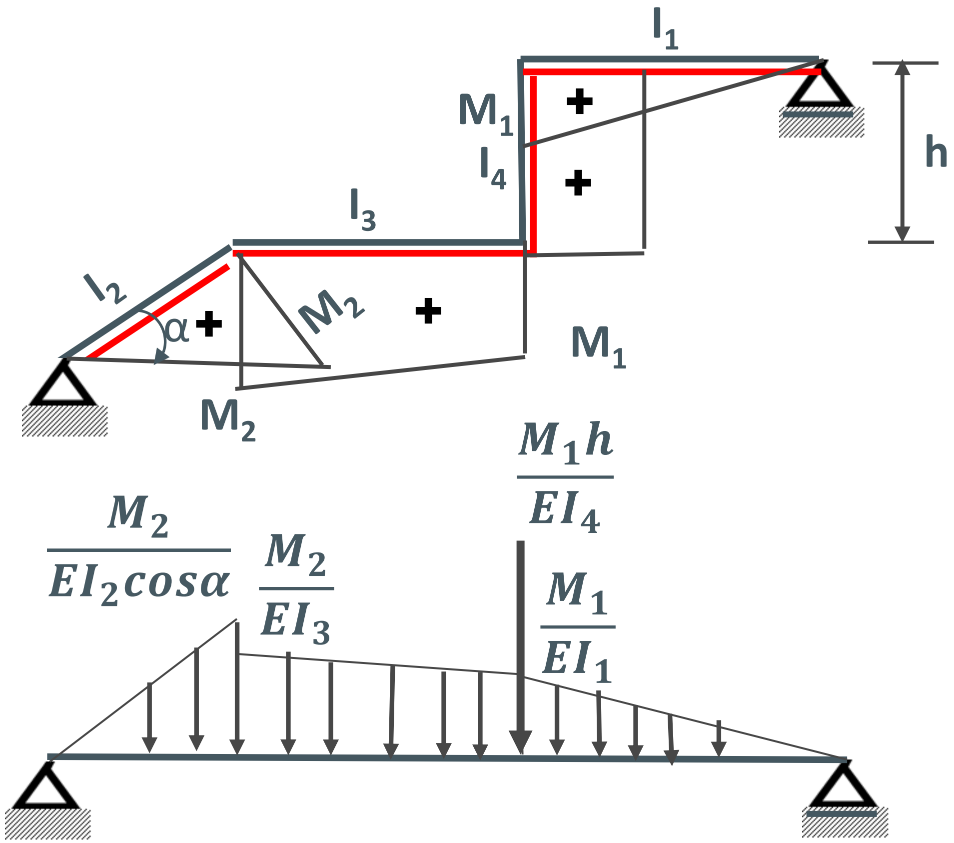
3. Sadece yörünge üzerindeki ***M(x)*** momentleri göz önüne alınır.

a. Yörünge üzerindeki çubuk yatay ise fiktif yük $\frac{M}{EI}$ olarak alınır.

b. Yörünge üzerindeki çubuk eğik ise fiktif yük $\frac{M}{EIcos\alpha}$ şeklinde alınır.

c. Yörünge üzerindeki çubuk düşey ise fiktif yük $\frac{M}{EI}$ diyagramının alanı olarak hesaplanır ve tekil yük olarak sisteme yüklenir.

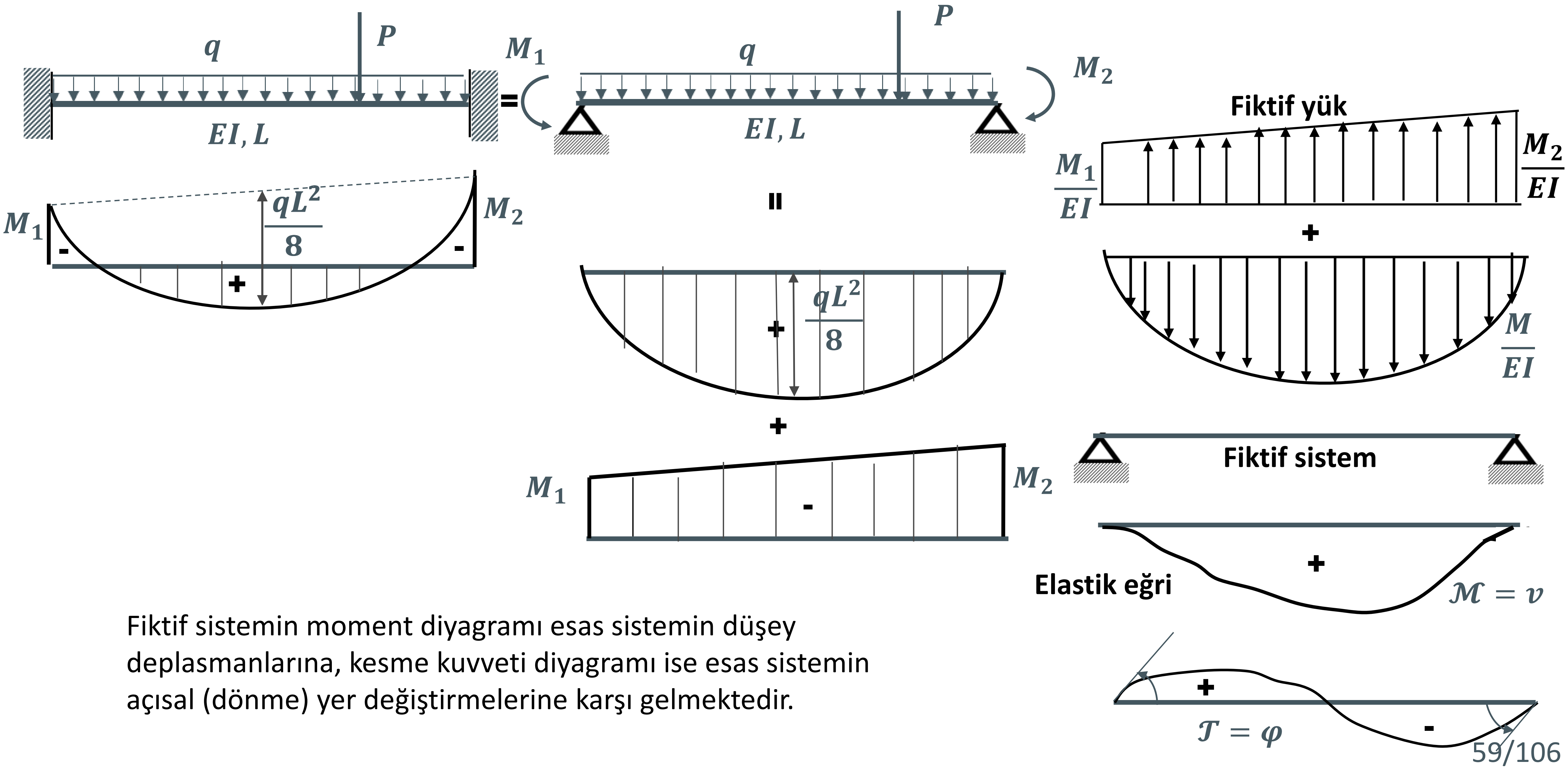
d. Moment pozitif ise fiktif yük yukarıdan aşağıya doğru negatif ise aşağıdan yukarı doğru yüklenir.



Gerçek

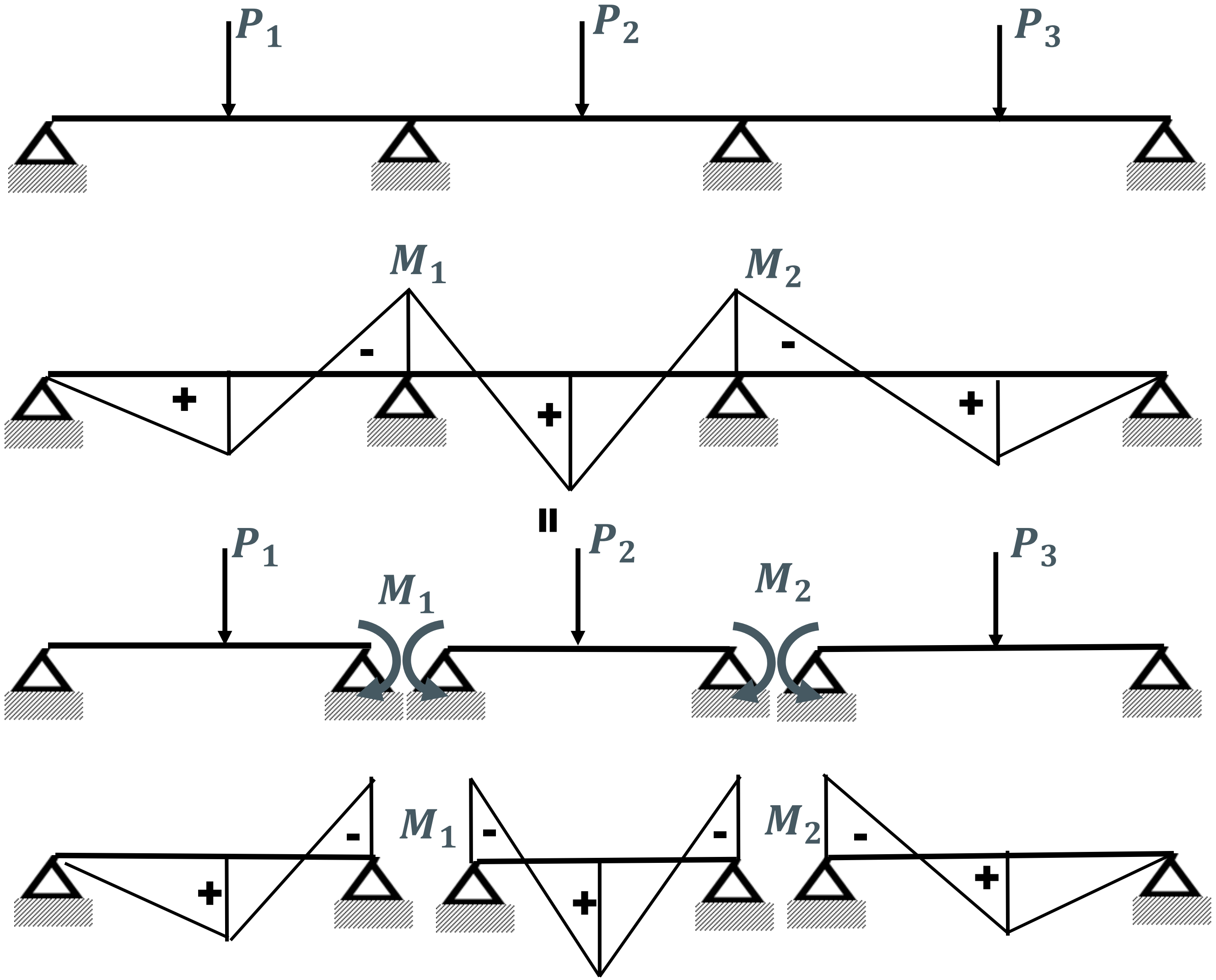
Fiktif

2. MOHR METODU İLE DEPLASMAN HESABI

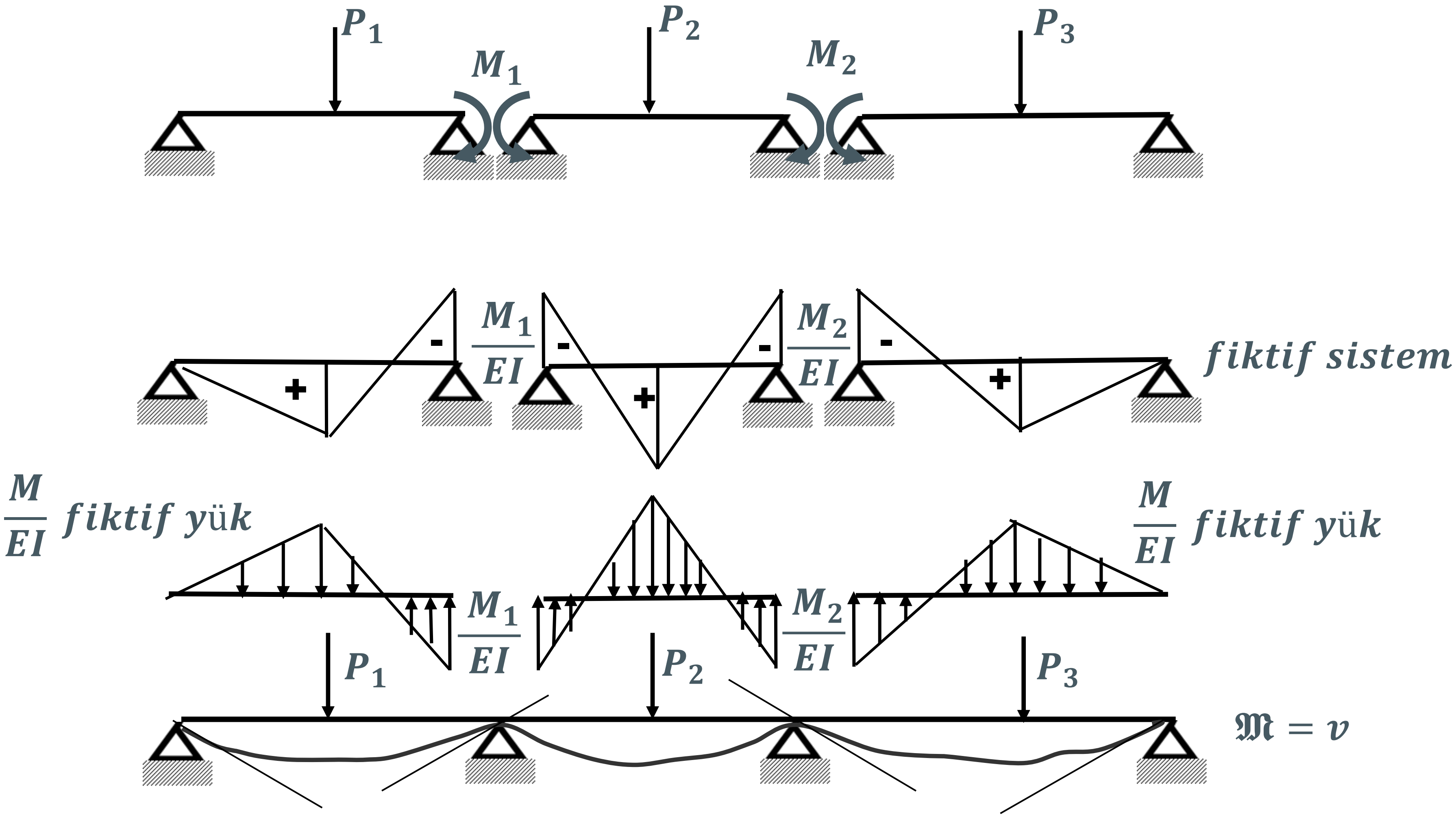


Fiktif sistemin moment diyagramı esas sistemin düşey deplasmanlarına, kesme kuvveti diyagramı ise esas sistemin açısal (dönme) yer değiştirmelerine karşı gelmektedir.

Sürekli Kiriş Mohr Yöntemi ile Düşey Deplasman Hesabı

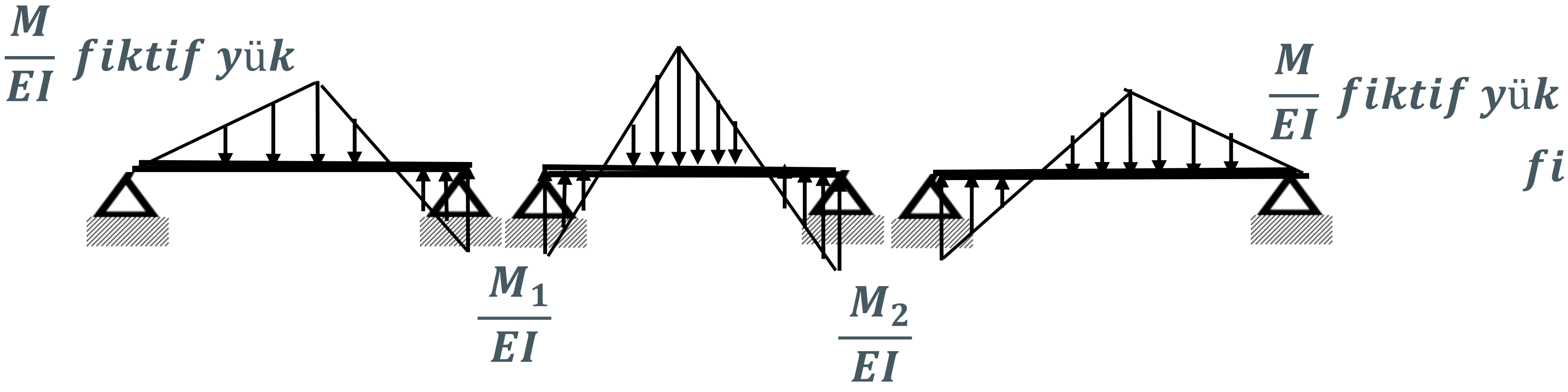


M $moment$ $diyagramı$

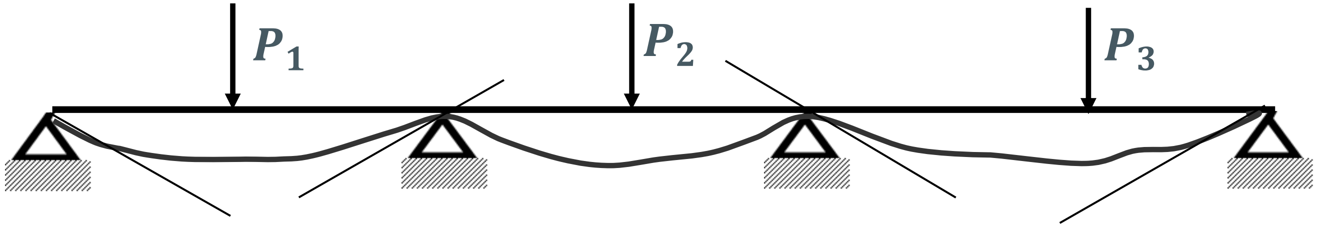


$\mathfrak{M} =$ *fiktif sistemin moment diyagramı*

$v =$ *esas sistemin deplasmanları*



fiktif sistem



$\mathfrak{M} = v$

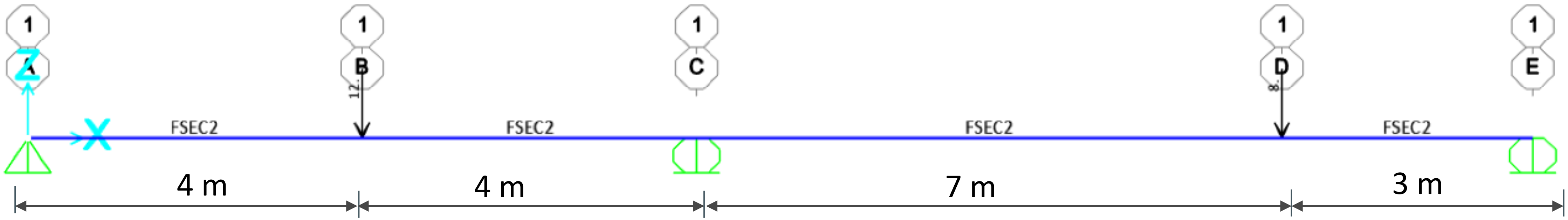
$\mathfrak{M} =$ *fiktif sistemin moment diyagramı*

$v =$ *esas sistemin deplasmanları*

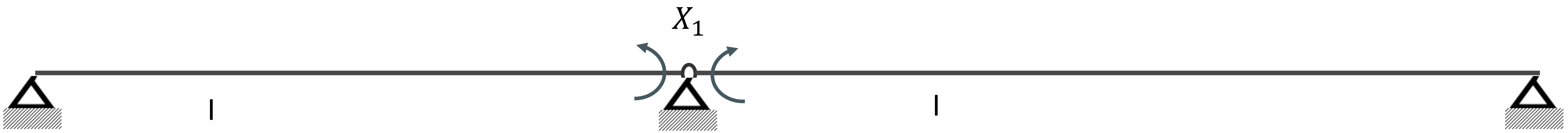
Örnek 12 Sürekli kiriş (Dış yükler altında çözümü)

M T diyagramlarını çiziniz

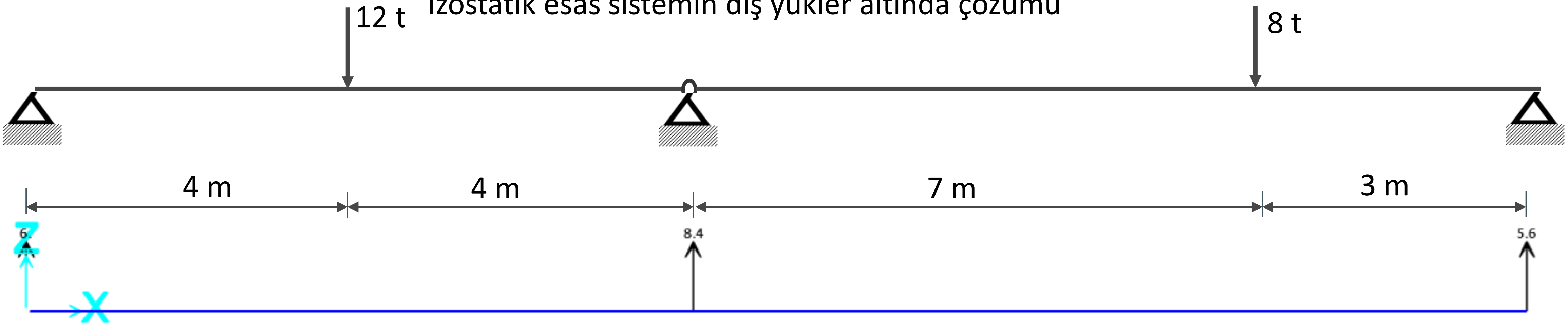
δ_B düşey deplasman, φ_A ve φ_C düşey dönmelerini Mohr yöntemi ile bulunuz.



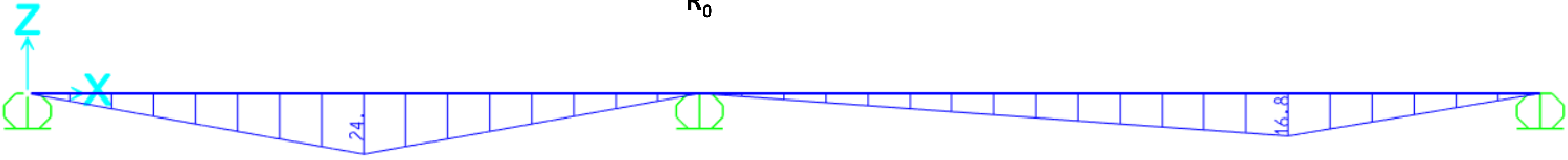
İzostatik esas sistem



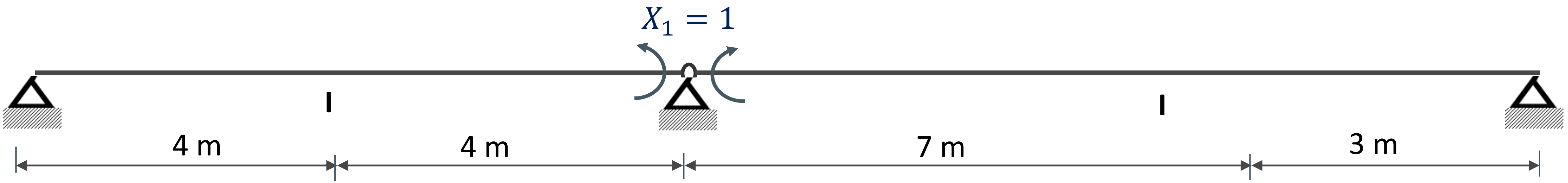
İzostatik esas sistemin dış yükler altında çözümü



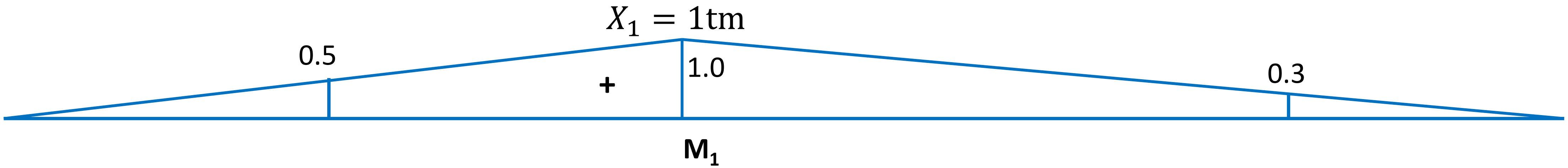
R_0



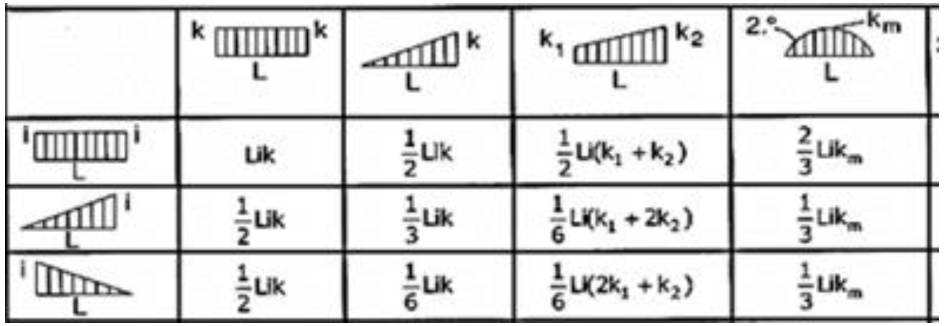
M_0



$X_1 = 1$ *birim yüklemesi altında moment diyagramı*

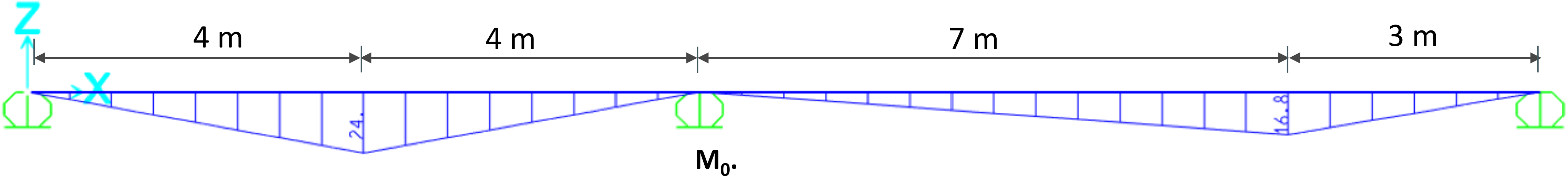


$$EI_c\delta_{11} = \frac{1}{3}8 * 1 * 1 * 1 + \frac{1}{3}10 * 1 * 1 * 1 = 6$$

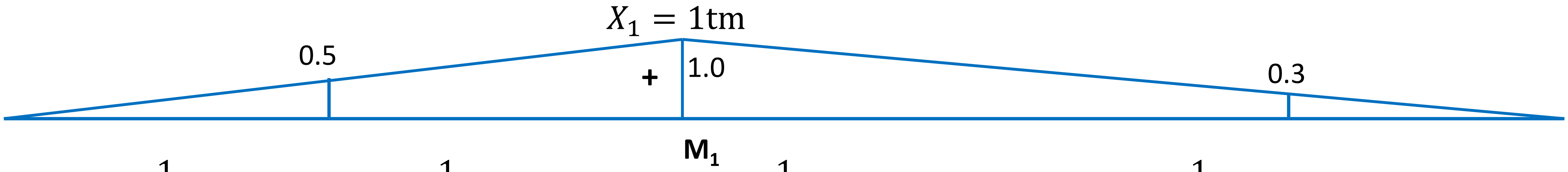


	k L k	L k	k₁ L k₂	2.° k_m L
i L i	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1 + k_2)	$\frac{2}{3}$Lik$_m$
L i	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1 + 2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k_1 + k_2)	$\frac{1}{3}$Lik$_m$

İzostatik esas sistemin dış yükler altında moment diyagramı



$X_1 = 1\ birim\ yüklemesi\ altında\ moment\ diyagramı$



$$EI_c\delta_{10} = \frac{1}{3}4 * 0.5 * 24 * 1 + \frac{1}{6}4 * 24(2 * 0.5 + 1) + \frac{1}{6}7 * 16.8(2 * 0.3 + 1) * 1 + \frac{1}{3}3 * 16.8 * 0.3 * 1 = 84.4$$

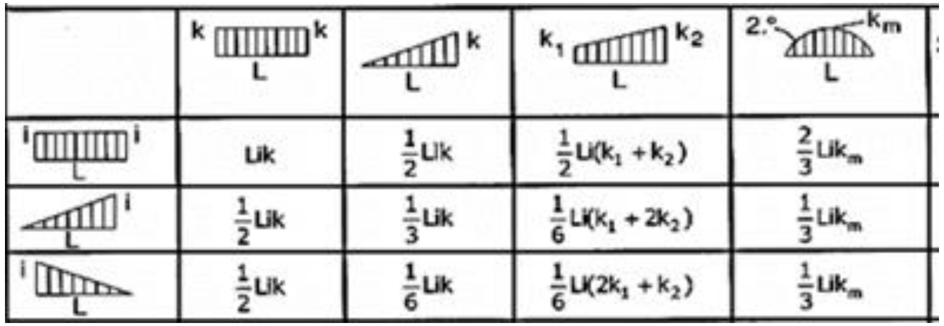
$$EI_c\delta_{11}X_1 + EI_c\delta_{10} = 0$$

$$6X_1 = -84.4 \quad \rightarrow \quad X_1 = -14.06\ tm \quad M = M_0 + M_1X_1$$

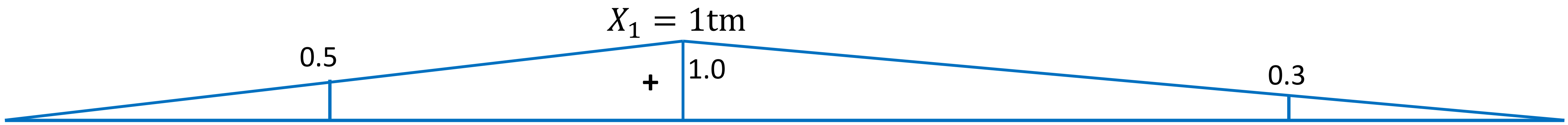
$$M_{(B)} = 24 + 0.5 * (-14.06) = 16.97\ tm$$

$$M_{(C)} = 0 + 1.0 * (-14.06) = -14.06\ tm$$

$$M_{(D)} = 16.8 + 0.3 * (-14.06) = 12.58\ tm$$

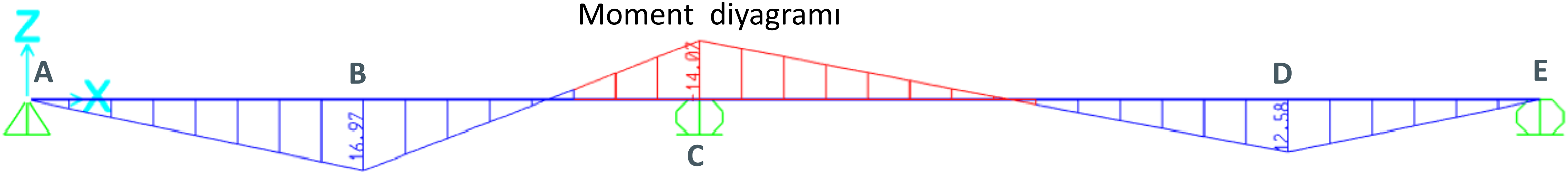


	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li$(2k_1+k_2)$	$\frac{1}{3}$Lik$_m$



M_1

Moment diyagramı



$$EI_c\delta_{11}X_1 + EI_c\delta_{10} = 0$$

$$6X_1 = -84.4 \quad \rightarrow \quad X_1 = -14.06\ tm \quad M = M_0 + M_1X_1$$

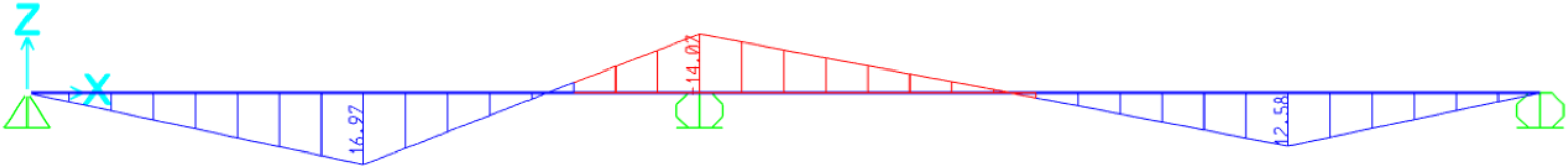
$$M_{(B)} = 24 + 0.5 * (-14.06) = 16.97\ tm$$

$$M_{(C)} = 0 + 1.0 * (-14.06) = -14.06\ tm$$

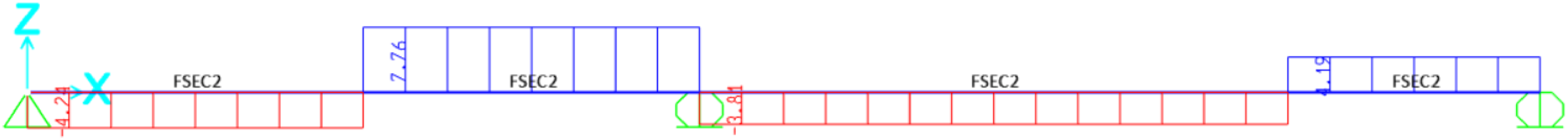
$$M_{(D)} = 16.8 + 0.3 * (-14.06) = 12.58\ tm$$

R mesnet tepkileri

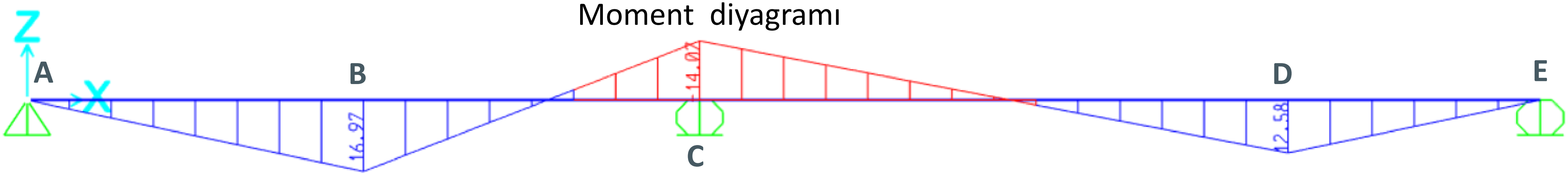
Moment diyagramı



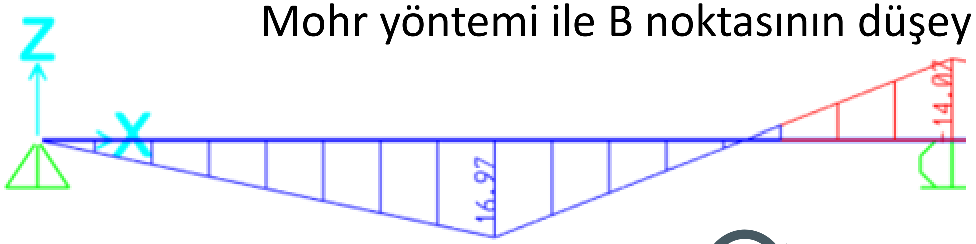
Kesme kuvveti diyagramı



Moment diyagramı

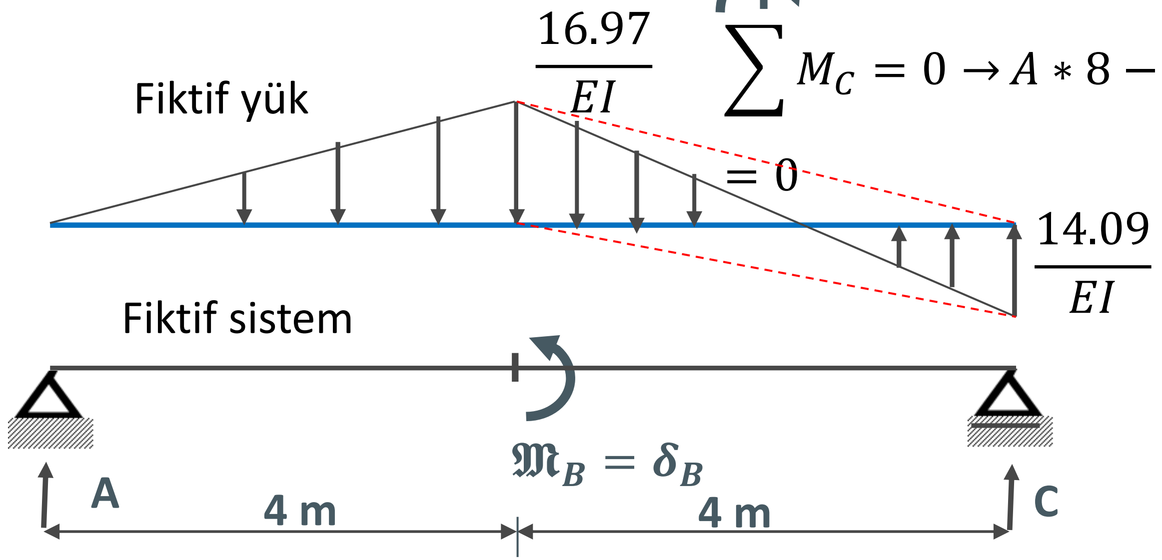


Mohr yöntemi ile B noktasının düşey deplasman hesabı $\boldsymbol{\delta_B}$ **= ?**



Fiktif yük

Fiktif sistem



$$A = \frac{29.25}{EI}$$

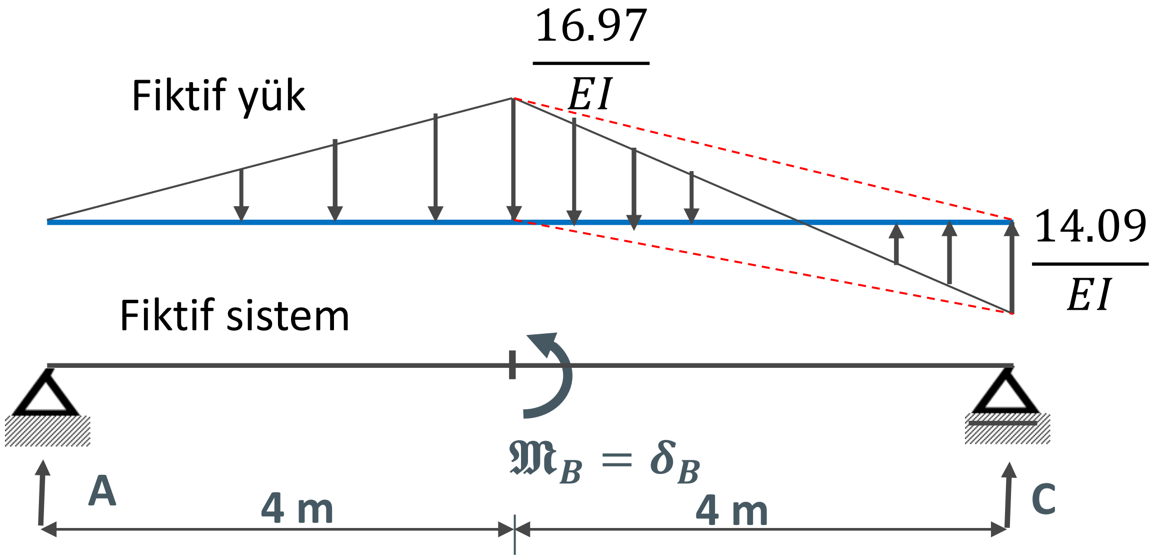
$$\boldsymbol{\delta_B} = \frac{71.7466}{EI}$$

$$\sum M_C = 0 \rightarrow A*8 - \frac{16.97}{EI}\frac{1}{2}4\left(\frac{1}{3}4+4\right) - \frac{16.97}{EI}\frac{1}{2}4*\left(\frac{2}{3}4\right) + \frac{14.07}{EI}\frac{1}{2}4*\frac{1}{3}*4 = 0$$

$$8A - \frac{234}{EI} = 0 \rightarrow \boxed{A = \frac{29.25}{EI}}$$

$$\mathfrak{M}_B + \frac{16.97}{EI}\frac{1}{2}*4*\frac{1}{3}*4 - \frac{29.25}{EI}*4 = 0$$

$$\mathfrak{M}_B - \frac{71.7466}{EI} = 0 \rightarrow \mathfrak{M}_B = \frac{71.7466}{EI} = \boldsymbol{\delta_B}$$



$$A = \frac{29.25}{EI} = \varphi_A$$

$$C = \frac{10.49}{EI} = \varphi_C$$

$$\uparrow + \sum y = 0$$

$$A - \frac{16.97}{EI}\frac{1}{2}4 - \frac{16.97}{EI}\frac{1}{2}4 + \frac{14.07}{EI}\frac{1}{2}4 + C = 0$$

$$C = -\frac{29.25}{EI} + \frac{33.94}{EI} + \frac{33.94}{EI} - \frac{28.14}{EI} = \frac{10.49}{EI}$$

RİJİTLİK MATRİSİ YÖNTEMİ İLE ÇÖZÜM (SDB88 PROGRAM*)

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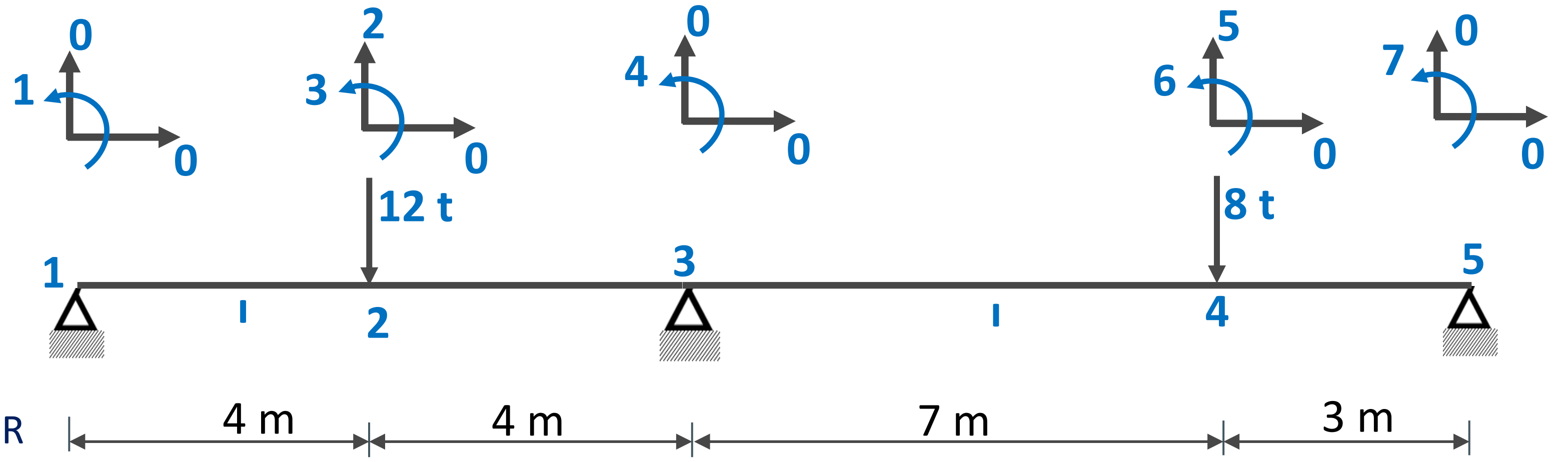
6. DUZLEMI ICERISINDE YUKLU GENEL CERCEVELERIN STATIK HESABI :
RİJİTLİK MATRİSİ YÖNTEMİ
2 AÇIKLIKLI KİRİŞ ÖRNEĞİ

ELEMAN SAYISI -------------= 4
DEPLASMAN SAYISI --------= 7
DUGUM SAYISI --------------= 5
ELASTISITE MODULU ------= 1
YUKLEME SAYISI ------------= 1
KAYMA DEFORMASYONLARI IHMAL EDILIYOR
```



```
   DUGUM     X      Y              DEPLASMAN NO              DEPLASMAN
   -------  ------  ------        ------------------------  ------------------
     1      0.00   0.00                 1                    -29.24444
     2      4.00   0.00                 2                    -71.73333
     3      8.00   0.00                 3                      4.68889
     4     15.00   0.00                 4                     10.48889
     5     18.00   0.00                 5                    -53.59668
                                        6                      5.28556
                                        7                     24.15556
```

$$A = \frac{29.25}{EI} = \varphi_A$$

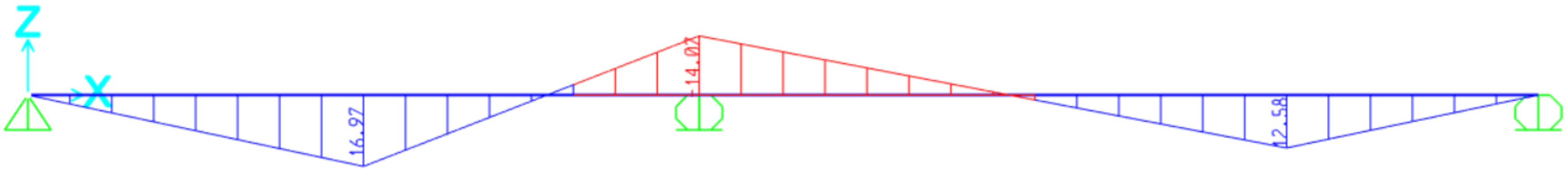
$$\boldsymbol{\delta_B} = \frac{71.7466}{EI}$$

$$C = \frac{10.49}{EI} = \varphi_C$$

*Dündar, C., Kıral, E., Mengi, Y., Yapı Mekaniğinde Bilgisayar Programları, Genişletilmiş 3. Baskı, Teknik Yayınevi, 1987.

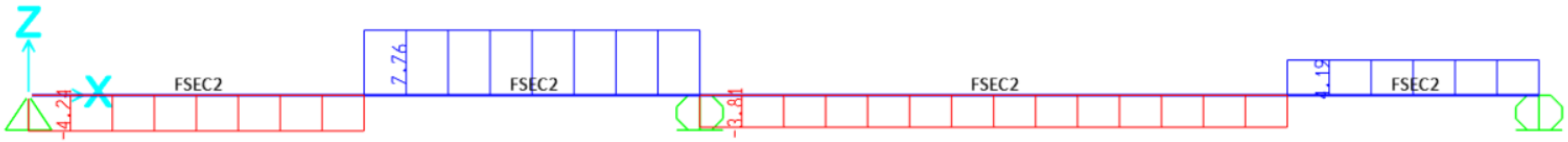
```
ELEMAN   i   j   BOYU    ALAN     ATALET     K O D  N U M A R A L A R I
---------------------------------------------------------------------------
   1     1   2   4.00   1.000    1.0000    0   0   1    0   2   3
   2     2   3   4.00   1.000    1.0000    0   2   3    0   0   4
   3     3   4   7.00   1.000    1.0000    0   0   4    0   5   6
   4     4   5   3.00   1.000    1.0000    0   5   6    0   0   7
YUKLEME NO = 1
```

Moment diyagramı

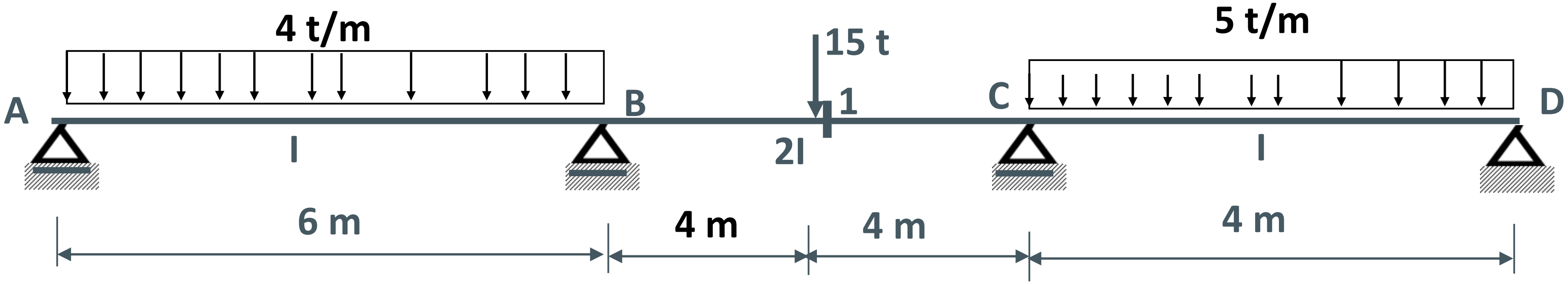


```
                    UC KUVVETLERI
                    ----------------------
 ELEMAN    Mij      Mji      Tij      Tji      Nj    ACIKLIK M.
 ------------------------------------------------------------------
 1        -0.00    16.97     4.24    -4.24    0.00
 2       -16.97   -14.07    -7.76     7.76    0.00
 3        14.07    12.58     3.81    -3.81    0.00
 4       -12.58     0.00    -4.19     4.19    0.00
```

Kesme kuvveti diyagramı



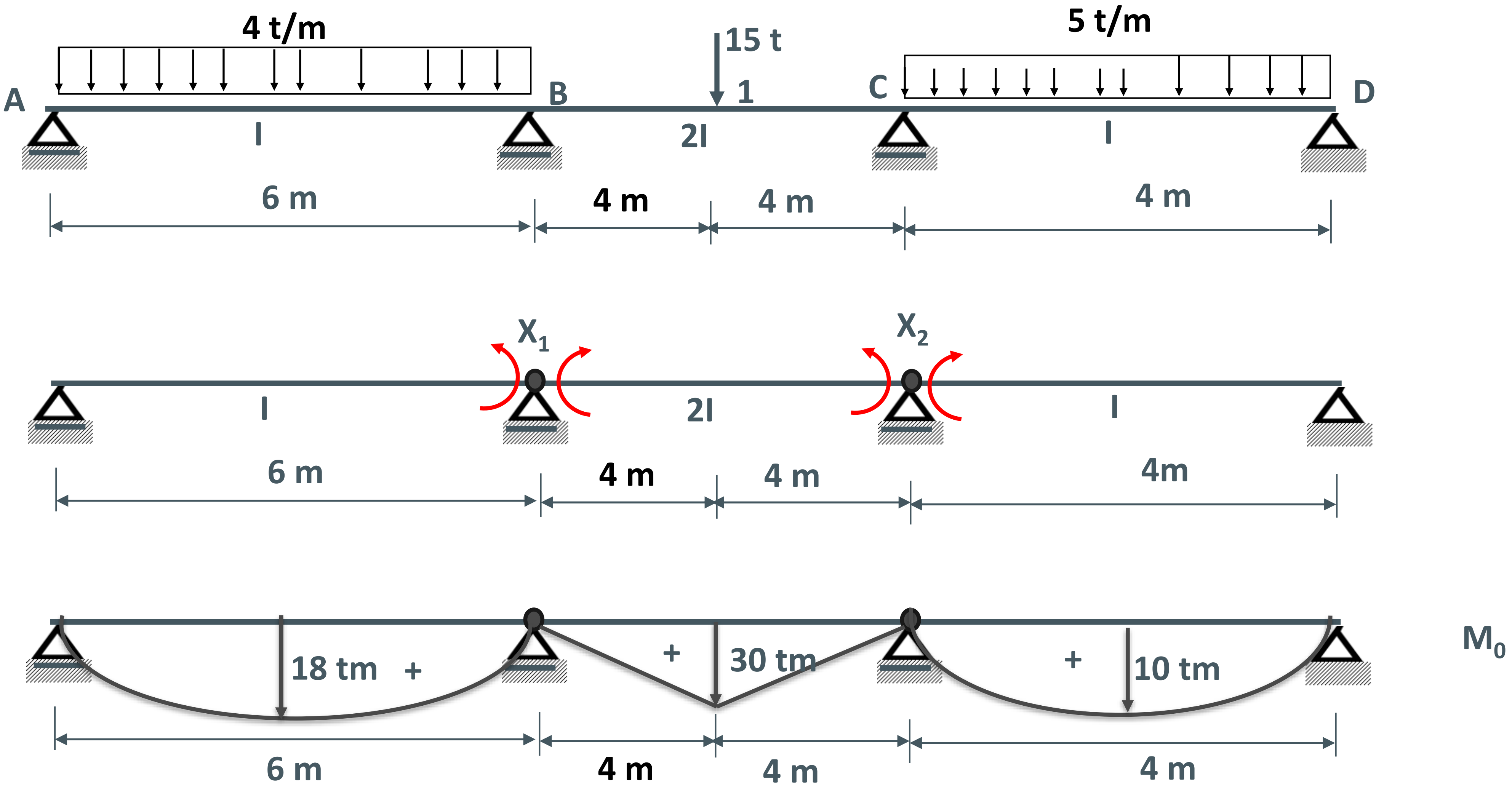
ÖRNEK 1

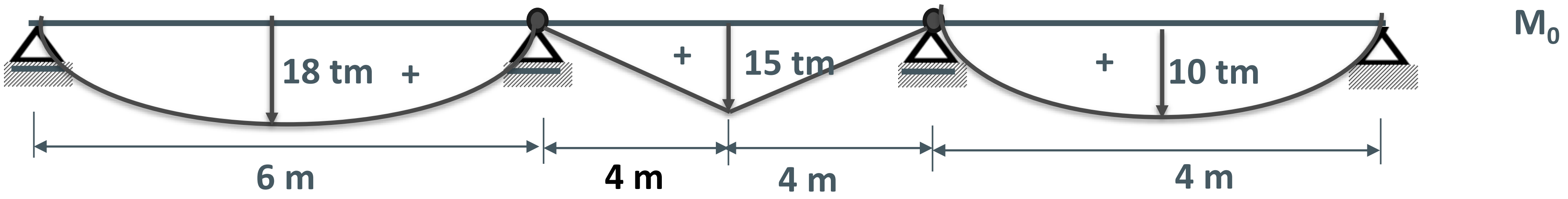


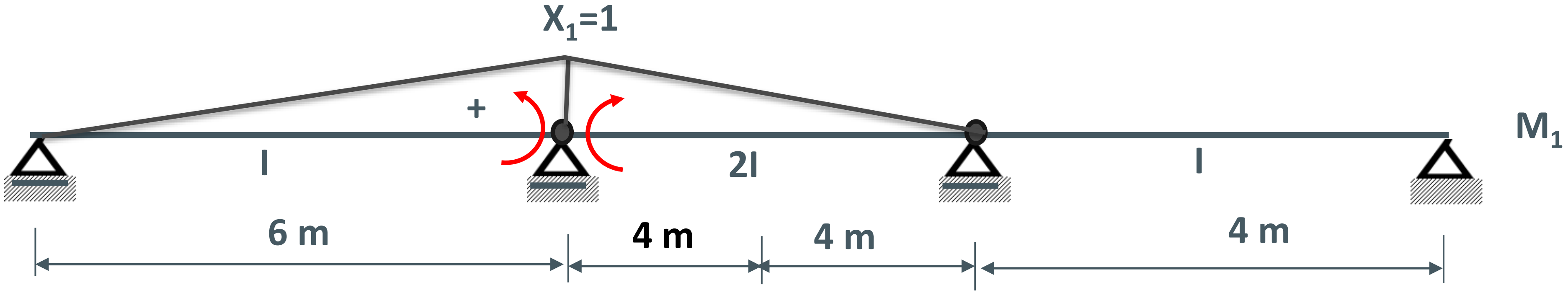
Şekilde verilen sürekli kirişin;

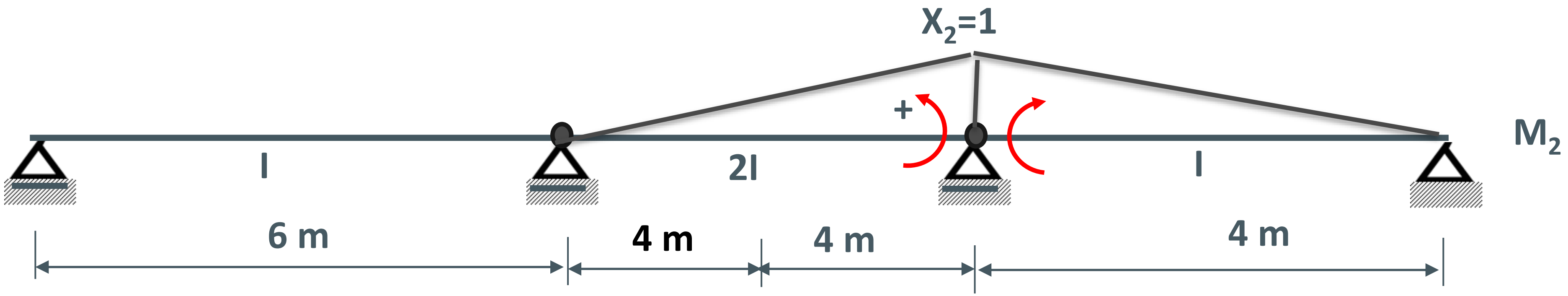
a. Moment ve kesme kuvveti diyagramlarını kuvvet yöntemini kullanarak çiziniz.

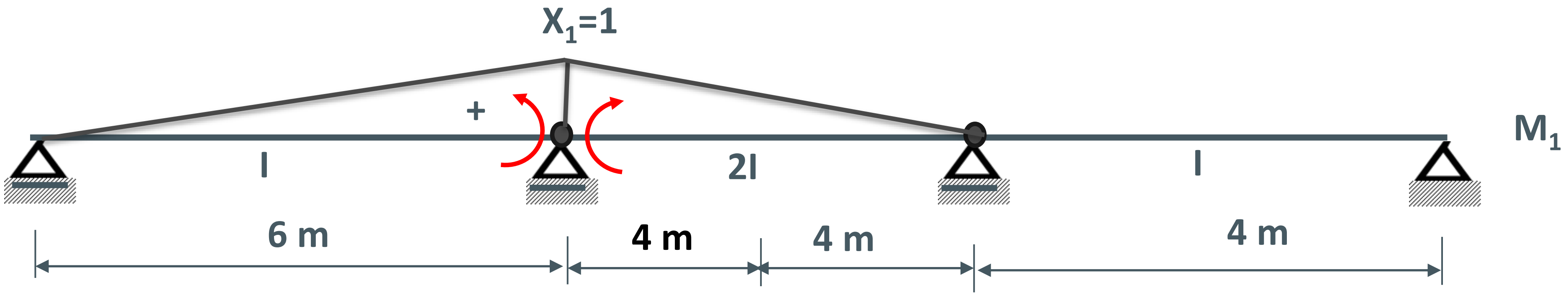
a. Kirişin BC açıklık ortasında 1 noktasının düşey deplasmanını (δ_1) ve C noktasının dönmesini (φ_C) Mohr Yöntemini kullanarak hesaplayınız.

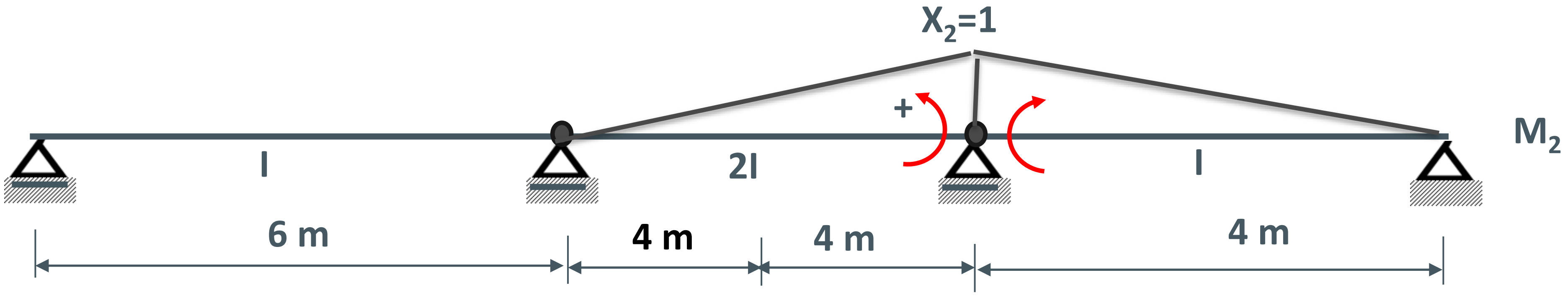

4 t/m
15 t
5 t/m
A
B
1
C
D
I
2I
I
6 m
4 m
4 m
4 m
X_1
X_2
I
2I
I
6 m
4 m
4 m
4m
18 tm
+
+
30 tm
+
10 tm
M_0
6 m
4 m
4 m
4 m


18 tm
+
+
15 tm
+
10 tm
6 m
4 m
4 m
4 m
M_0


X_1=1
+
I
2I
I
6 m
4 m
4 m
4 m
M_1


X_2=1
+
I
2I
I
6 m
4 m
4 m
4 m
M_2

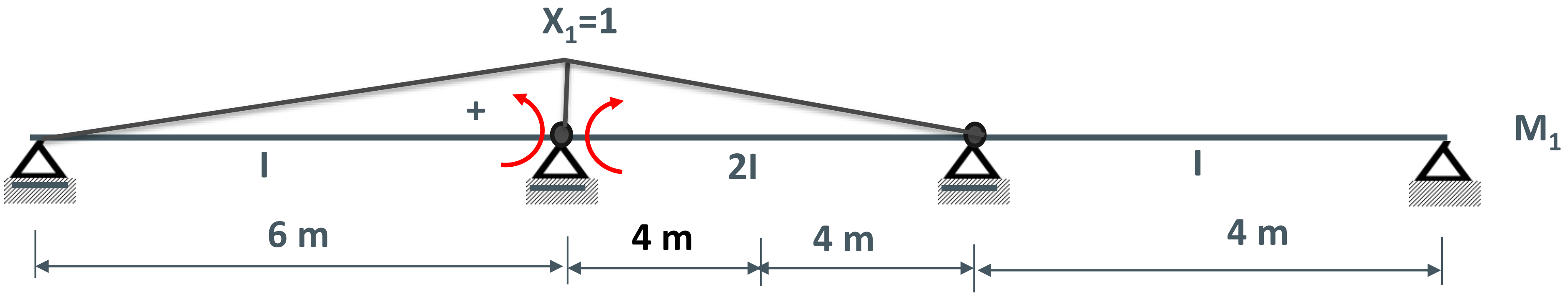


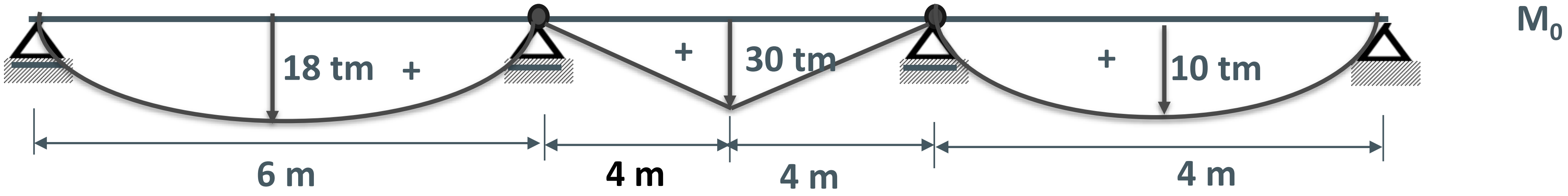


$$EI_c\delta_{11} = \frac{1}{3}Lik[\frac{I_C}{I}] = \frac{1}{3}6*1*1*[2] + \frac{1}{3}8*1*1*[1] = 6.67$$

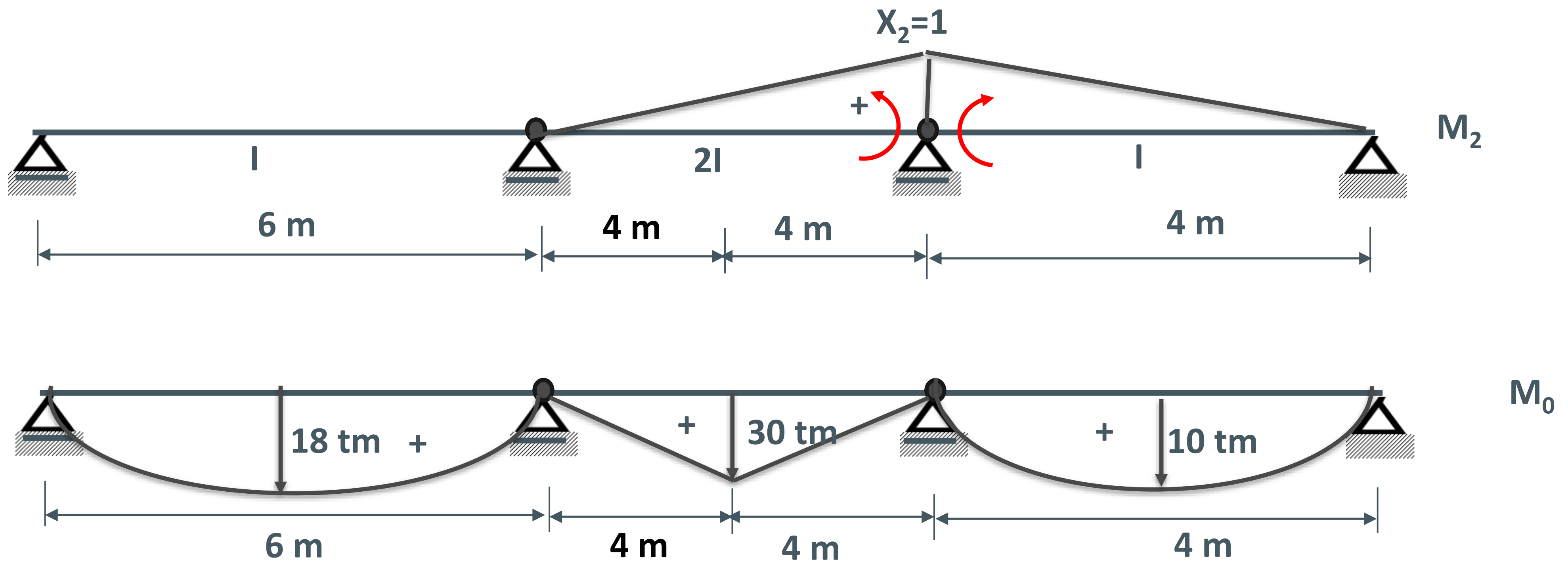
$$EI_c\delta_{12} = \frac{1}{6}Lik[\frac{I_C}{I}] = \frac{1}{6}8*1*1*[1] = 1.33$$

$$EI_c\delta_{22} = \frac{1}{3}Lik[\frac{I_C}{I}] = \frac{1}{3}8*1*1*[1] + \frac{1}{3}4*1*1*[2] = 5.33$$





$$EI_c\delta_{10} = \frac{1}{3}Lik_m[\frac{I_c}{I}] + \frac{1}{6}L(1+\beta)ik[\frac{I_c}{I}] = \frac{1}{3}6*1*18*[2] + \frac{1}{6}8(1+0.5)*1*30*[1] = 132$$

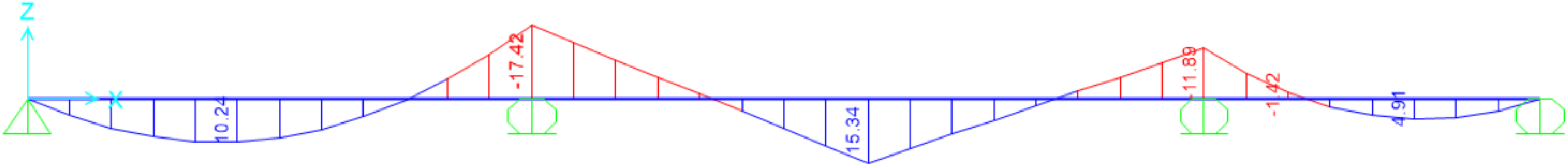


$$EI_c\delta_{20} = \frac{1}{6}L(1+\beta)ik\left[\frac{I_c}{I}\right] + \frac{1}{3}Lik_m\left[\frac{I_c}{I}\right] = \frac{1}{6}8(1+0.5)*1*30*[1] + \frac{1}{3}4*1*10*[2] = 86.67$$

$$6.67X_1 + 1.33X_2 = 132$$
$$1.33X_1 + 5.33X_2 = 86.67$$

$$X_1 = -17.414\ tm \quad X_2 = -11.915\ tm$$

$$M = -17.414M_1 + (-11.915)M_2 \quad Moment\ Diyagramı$$

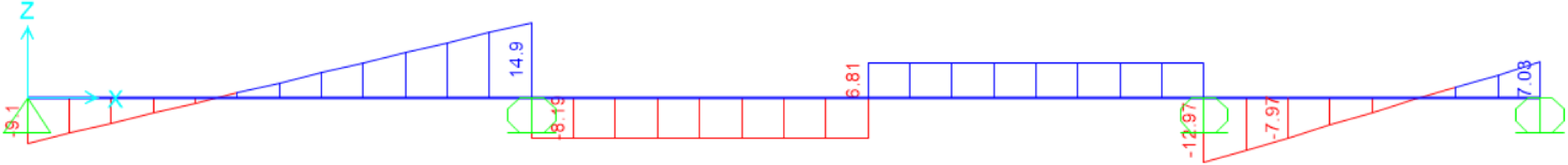


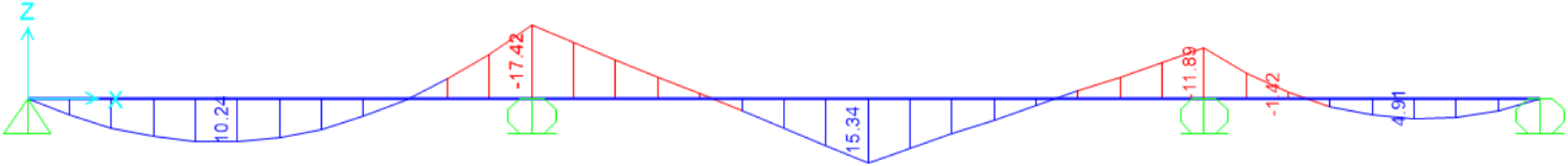
$$6.67X_1 + 1.33X_2 = 132$$

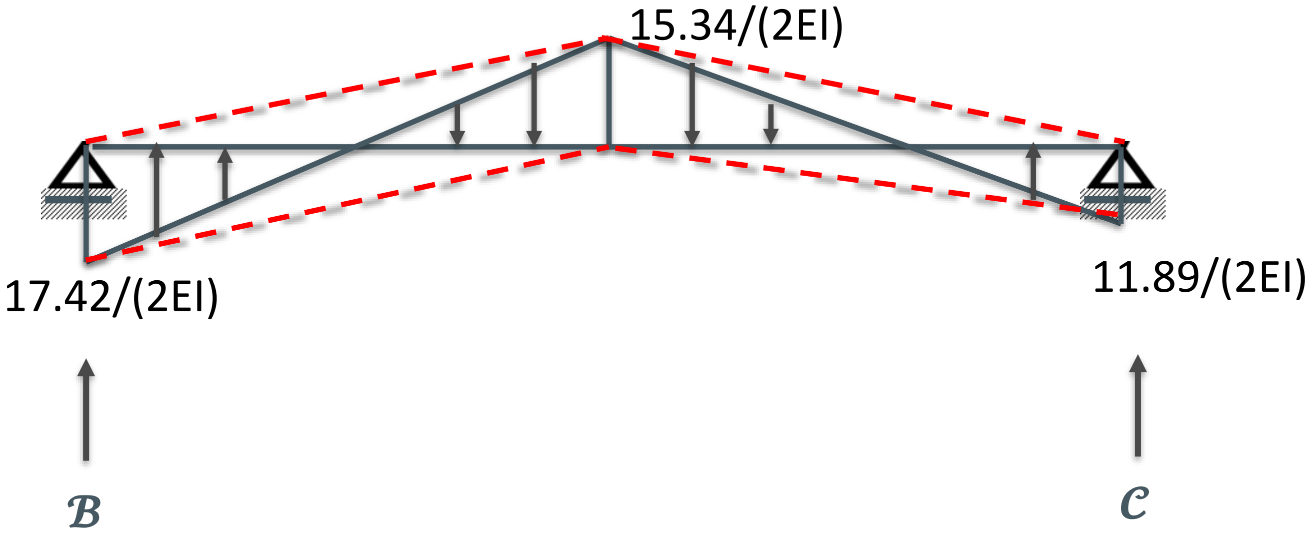
$$1.33X_1 + 5.33X_2 = 86.67$$

$$X_1 = -17.414\ tm \quad X_2 = -11.915\ tm$$

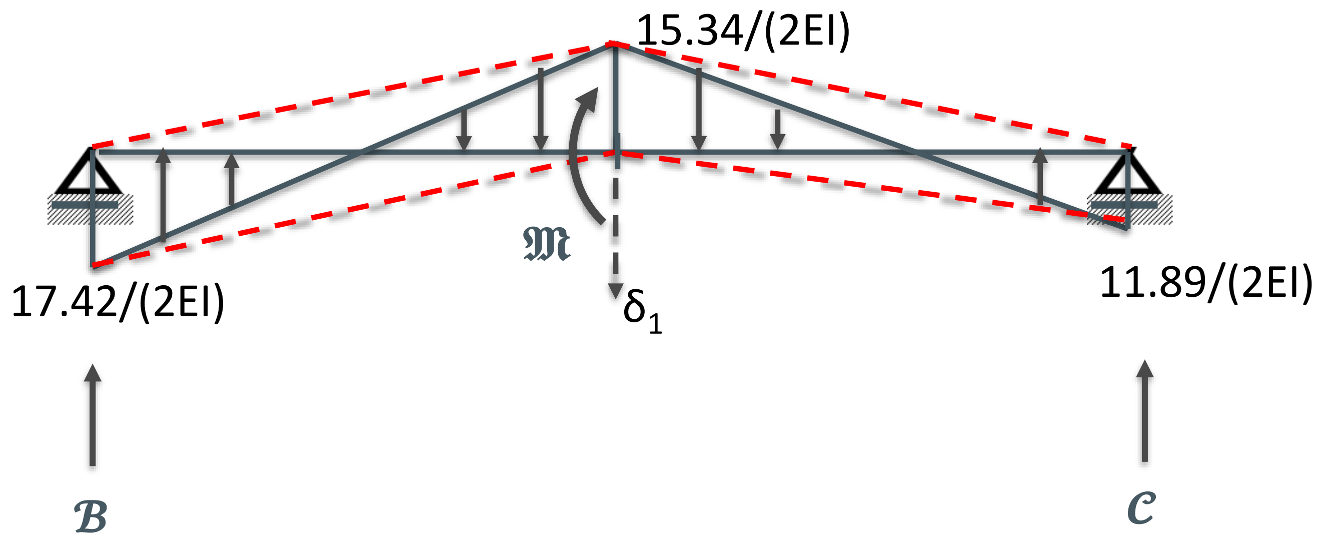
$T = -17.414T_1 + (-11.915)T_2$ *Kesme Kuvveti Diyagramı*







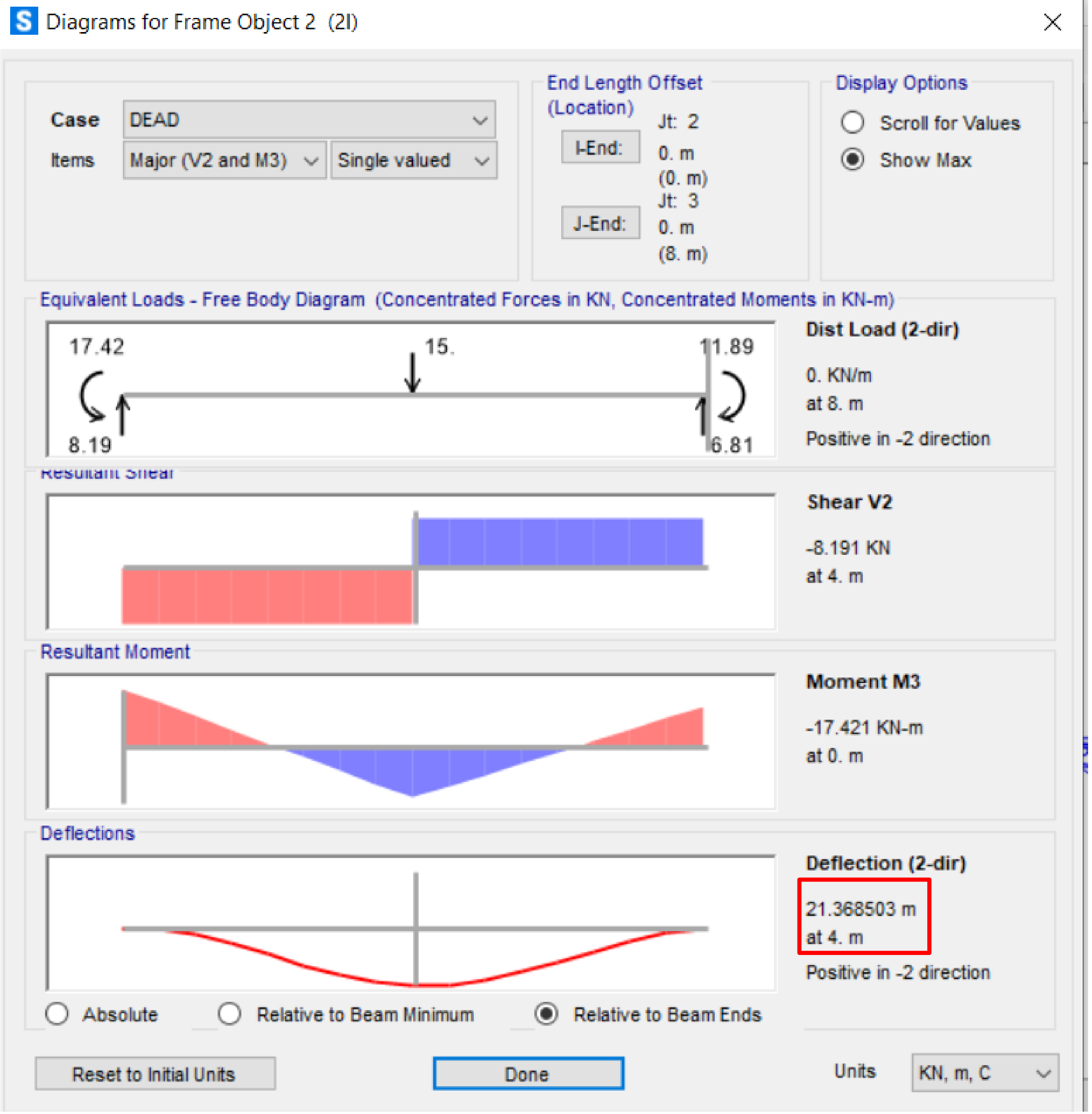
$$\sum M_B = 0 \rightarrow \boldsymbol{\mathcal{C}} * 8 + \frac{11.89}{2EI} * 4 * \frac{1}{2}\left(\frac{2}{3}4 + 4\right) - \frac{15.34}{2EI} * 4 * \frac{1}{2}\left(\frac{1}{3}4 + 4\right) + \frac{17.42}{2EI} * 4 * \frac{1}{2}\left(\frac{1}{3}4\right) - \frac{15.34}{2EI} * 4 * \frac{1}{2}\left(\frac{2}{3}4\right) = 0$$



$$\sum M_B = 0 \rightarrow \mathcal{C} * 8 + \frac{11.89}{2EI} * 4 * \frac{1}{2}\left(\frac{2}{3}4 + 4\right) - \frac{15.34}{2EI} * 4 * \frac{1}{2}\left(\frac{1}{3}4 + 4\right) + \frac{17.42}{2EI} * 4 * \frac{1}{2}\left(\frac{1}{3}4\right) - \frac{15.34}{2EI} * 4 * \frac{1}{2}\left(\frac{2}{3}4\right) = 0$$

$$8 * C - \frac{20.24}{EI} = 0 \rightarrow C = \varphi_C = \frac{20.24}{8EI} = \frac{2.53}{EI} rad$$

$$\sum \mathfrak{M}_1 = 0 \ \rightarrow \mathfrak{M}_1 - \frac{2.53}{EI} * 4 - \frac{11.89}{2EI} * 4 * \frac{1}{2} * \left(\frac{2}{3}4\right) + \frac{15.34}{2EI} * 4 * \frac{1}{2}\left(\frac{1}{3}4\right) = 0 \rightarrow \mathfrak{M}_1 = \delta_1 = \frac{21.37}{EI} \ m$$



Diagrams for Frame Object 2 (2I)

Case: DEAD

Items: Major (V2 and M3) — Single valued

End Length Offset (Location)

Jt: 2
I-End: 0. m
(0. m)

Jt: 3
J-End: 0. m
(8. m)

Display Options

○ Scroll for Values

● Show Max

Equivalent Loads - Free Body Diagram (Concentrated Forces in KN, Concentrated Moments in KN-m)

Dist Load (2-dir)
0. KN/m
at 8. m
Positive in -2 direction

Resultant Shear

Shear V2
-8.191 KN
at 4. m

Resultant Moment

Moment M3
-17.421 KN-m
at 0. m

Deflections

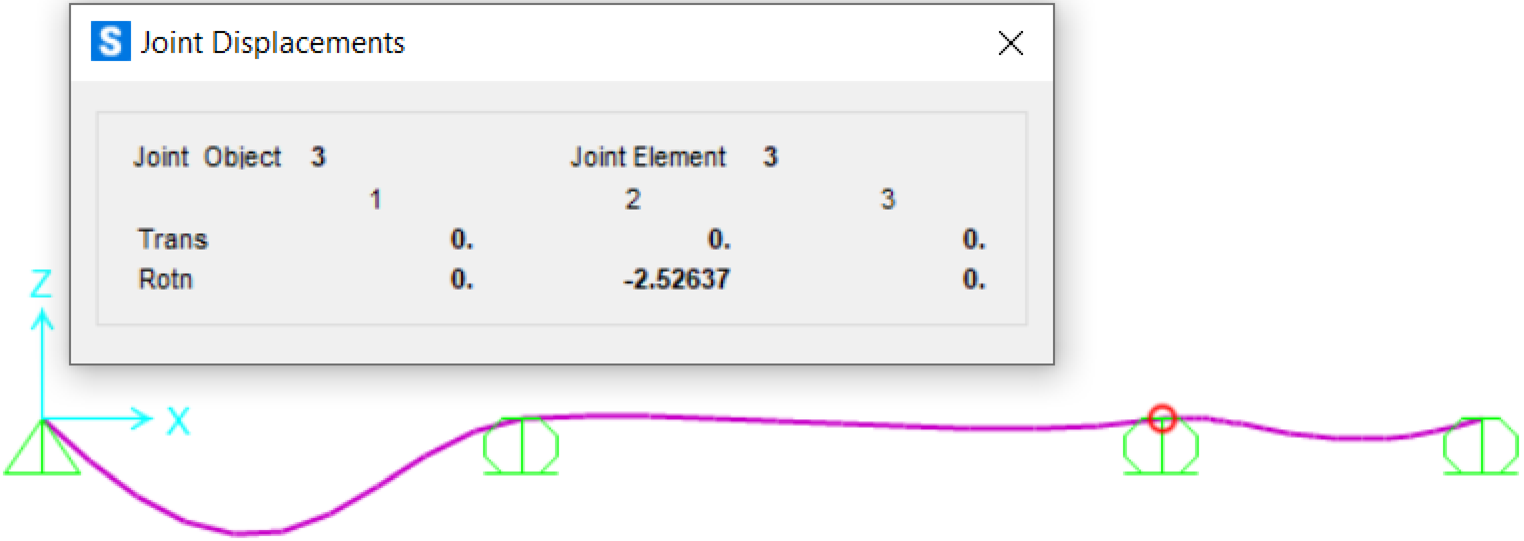
Deflection (2-dir)
21.368503 m
at 4. m
Positive in -2 direction

○ Absolute ○ Relative to Beam Minimum ● Relative to Beam Ends

Units: KN, m, C

$$\mathfrak{M}_1 = \delta_1 = \frac{21.37}{EI}\,\mathrm{m}$$

$$\boldsymbol{c} = \varphi_C = \frac{20.24}{8EI} = \frac{2.53}{EI} rad$$



Joint Displacements

```
Joint Object   3                    Joint Element   3
                   1                       2                  3
Trans                    0.                      0.                 0.
Rotn                     0.               -2.52637                  0.
```

ELASTİK MESNETLİ VE ELASTİK BİRLEŞİMLİ SİSTEMLER

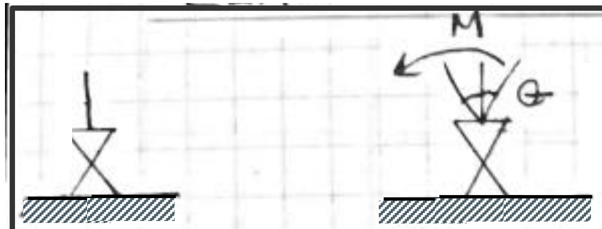
Düğüm noktalarında veya sistemin başka bir yerinde olan elastik bir birleşimde M momenti ile iki kenarın birbirine göre rölatif Δθ dönmesi arasındaki oran sabittir. Δu = Δv=0

Pozitif olan ve R_θ ile gösterilen bu sabite birleşim redörü denir.

Rijit birleşim $R_\theta = \infty$

Mafsallı birleşim $R_\theta = 0$

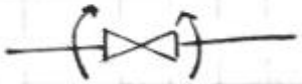
$0 < R_\theta < \infty$



$$\frac{M}{\theta} = R_\theta$$ Dönme redörü (mesnet redörü)

$R_\theta > 0$

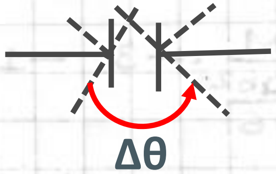
Dönmeye karşı elastik ankastre mesnet

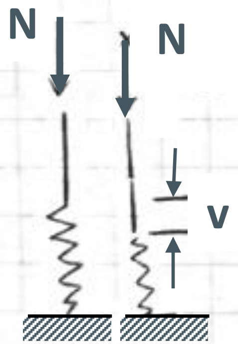


$$\frac{M}{\Delta\theta} = \overline{R}_\theta$$

Elastik birleşim redörü > 0 (sabit)

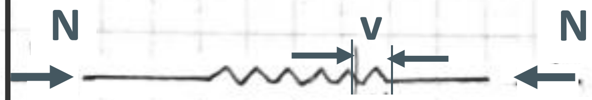
Dönmeye karşı elastik birleşim





$$R_v = \frac{N}{v} > 0 \text{ çökme redörü (sabit)}$$

Çökmeye karşı elastik ankastre mesnet



Çökmeye karşı elastik birleşim $\frac{N}{v} = \overline{R}_v > 0$

Dönmeye karşı elastik ankastre olan mesnette M momenti ile θ dönmesi arasındaki oran sabittir ve u=v=0 olur.

Pozitif olan ve R_θ ile gösterilen bu sabite mesnet redörü denir.

Ankastre mesnette $R_\theta = \infty$

Sabit mesnette $R_\theta = 0$

$0 < R_\theta < \infty$

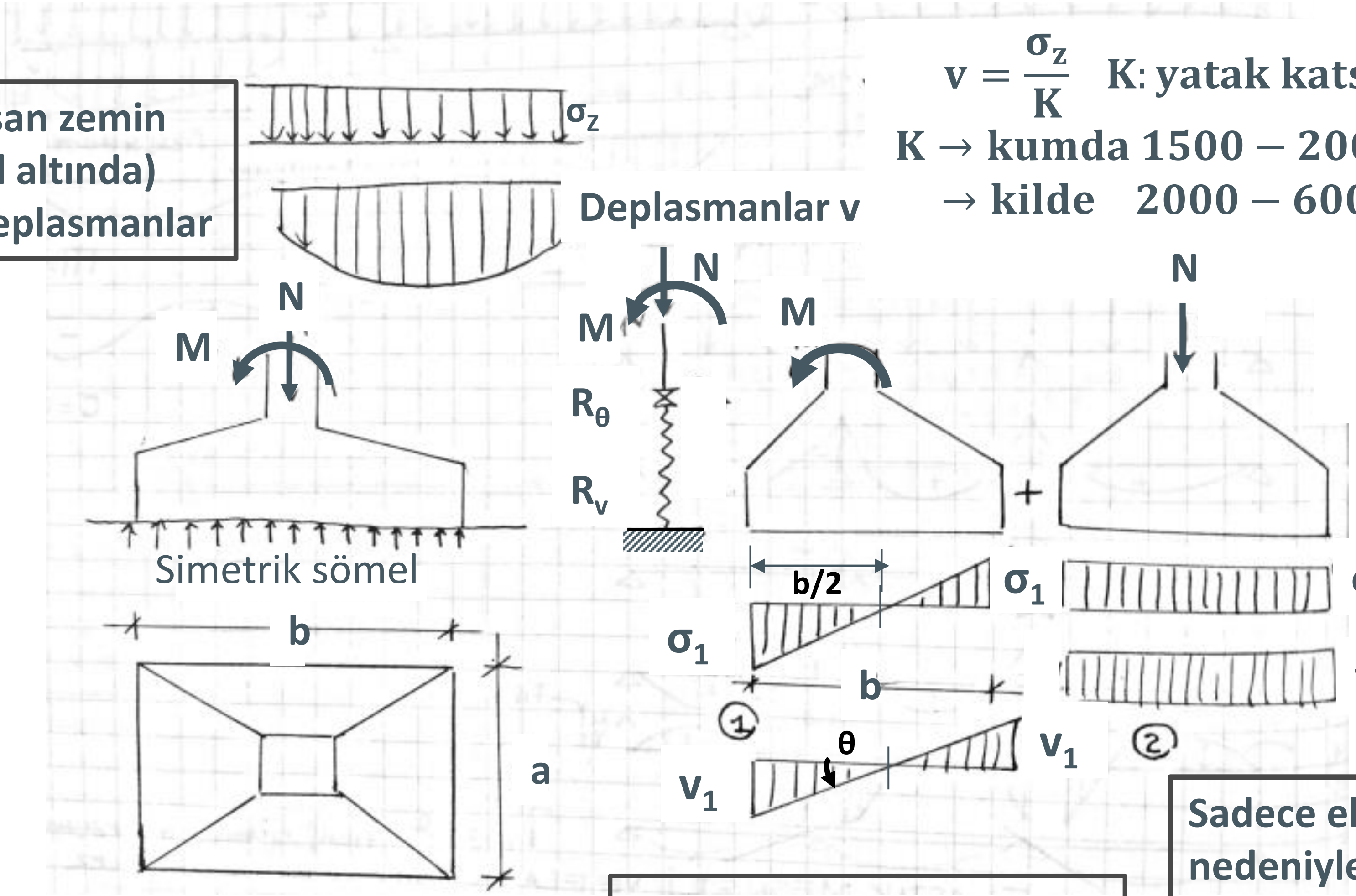
ELASTİK ZEMİNE OTURAN YAPILAR İÇİN MESNET REDÖRLERİNİN HESABI

σ_z : temel altında oluşan zemin gerilmeleri (rijit temel altında)
v : zeminde oluşan deplasmanlar

$$v = \frac{\sigma_z}{K} \quad K\text{: yatak katsayısı}$$

$$K \rightarrow \text{kumda } 1500 - 2000 \ t/m^3$$

$$\rightarrow \text{kilde} \quad 2000 - 6000 \ t/m^3$$



R_θ: Dönme redörü

R_v : Çökme redörü

Gerilme diyagramı

Çökme diyagramı

Moment nedeniyle oluşan Zemin gerilmeleri ve deplasmanlar

Sadece eksenel kuvvet nedeniyle oluşan üniform zemin gerilmesi ve deplasmanlar

Zeminin bir noktasındaki çökmenin o noktadaki gerilme ile orantılı olduğu kabul edilirse $\mathrm{v} = \frac{\sigma}{\mathrm{K}} \quad \mathrm{K} > 0 \;\; \mathrm{t/m^2}$ boyutunda olan K sabitine zeminin yatak katsayısı denir.

ÇEŞİTLİ ZEMİN TÜRLERİ İÇİN YAKLAŞIK K DEĞERLERİ

ZEMİN TÜRÜ	K ($\mathrm{t/m^3}$)
Balçık, Turba	<200
Kil, plastik	500-1000
Kil, yarı sert	1000-1500
Kil, sert	1500-3000
Dolma toprak	1000-2000
Kum, gevşek	1000-2000
Kum, orta sıkı	2000-5000
Kum, sıkı	5000-10000
Kum-çakıl, sıkı	10000-15000
Sağlam şist	>50000
Kaya	>200000

Zeminin bir noktasındaki çökmenin o noktadaki gerilme ile orantılı olduğu kabul edilirse $\mathrm{v} = \frac{\sigma}{K}$ $K > 0$ t/m^2 boyutunda olan K sabitine zeminin yatak katsayısı denir.

1 durumu: Temele sadece M momenti aktarıldığı zaman

$$\mathbf{v_1} = \frac{\sigma_1}{K} \qquad (1)$$

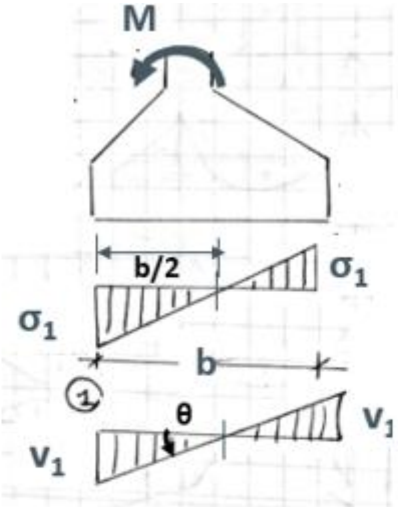
$$\theta = \frac{v_1}{b/2} \qquad (2)$$

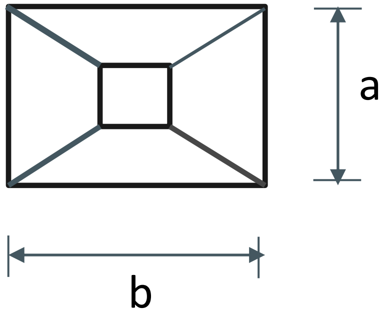
$$\sigma_1 = \frac{M}{I}y = \frac{M}{\frac{ab^3}{12}}\frac{b}{2} = \frac{M}{\frac{ab^2}{6}} \qquad (3)$$

$$v_1 = \frac{M}{\frac{ab^2}{6}K} \qquad (4)$$

$$\theta = \frac{M}{\frac{ab^3}{12}K} \qquad (5)$$

$$R_\theta = \frac{M}{\theta} = \frac{ab^3}{12}K$$



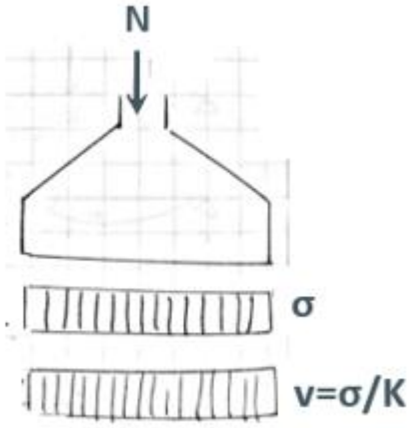


2 durumu: Temele yalnız N düşey kuvvet aktarıldığı zaman

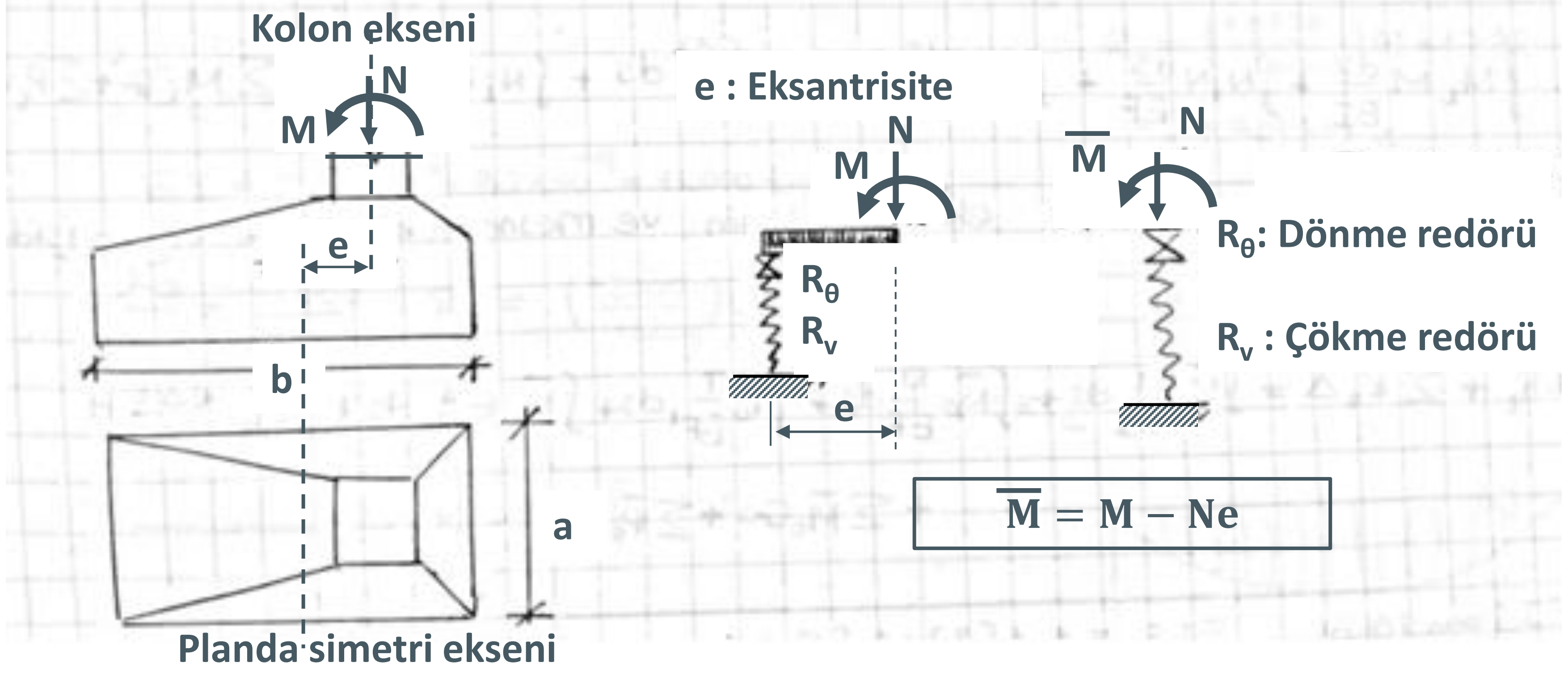
$$\sigma = \frac{N}{ab}$$

$$v = \frac{\sigma}{K} = \frac{N}{abK}$$

$$R_v = \frac{N}{v} = abK$$



SİMETRİK OLMAYAN SÖMELLER



e : Eksantrisite

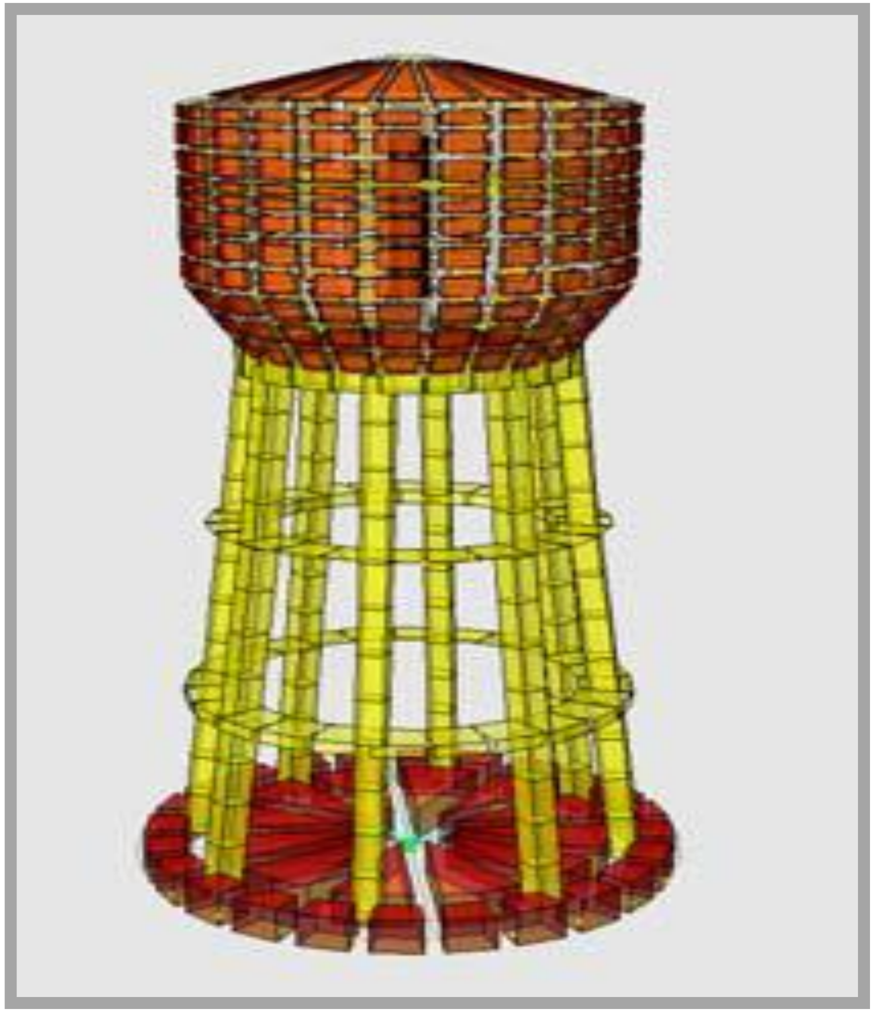
R_θ: Dönme redörü

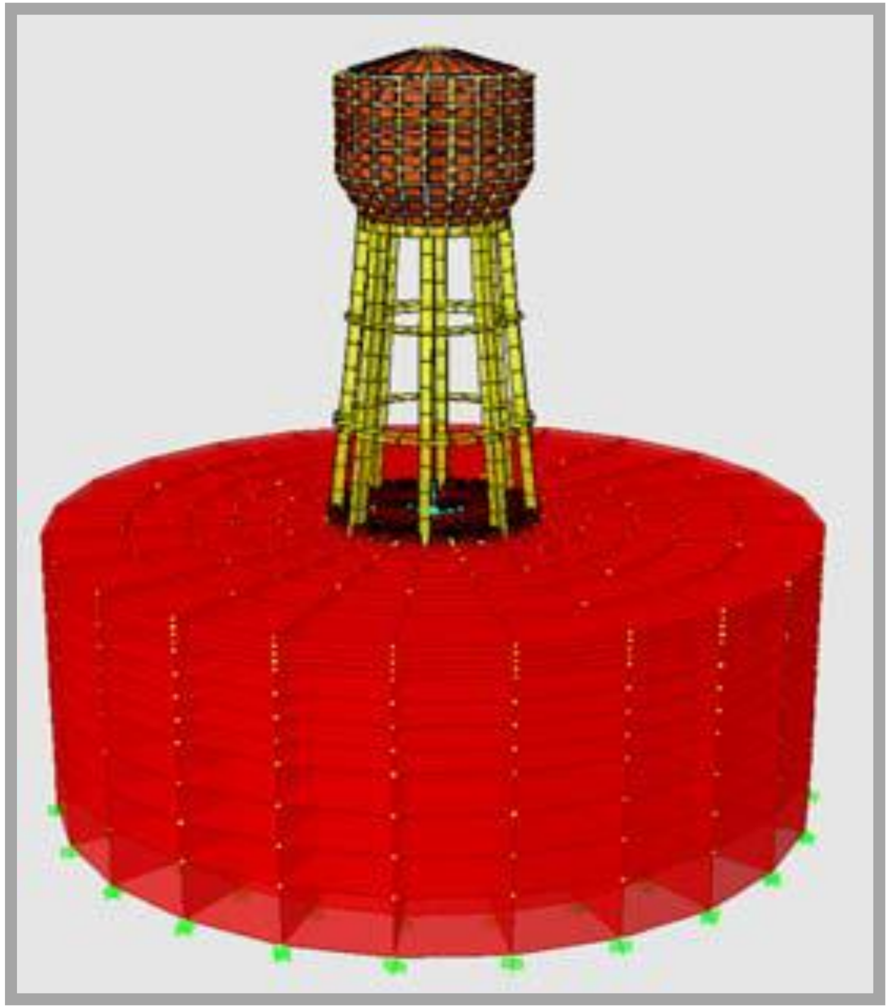
R_v : Çökme redörü

$$\boxed{\overline{\mathbf{M}} = \mathbf{M} - \mathbf{Ne}}$$

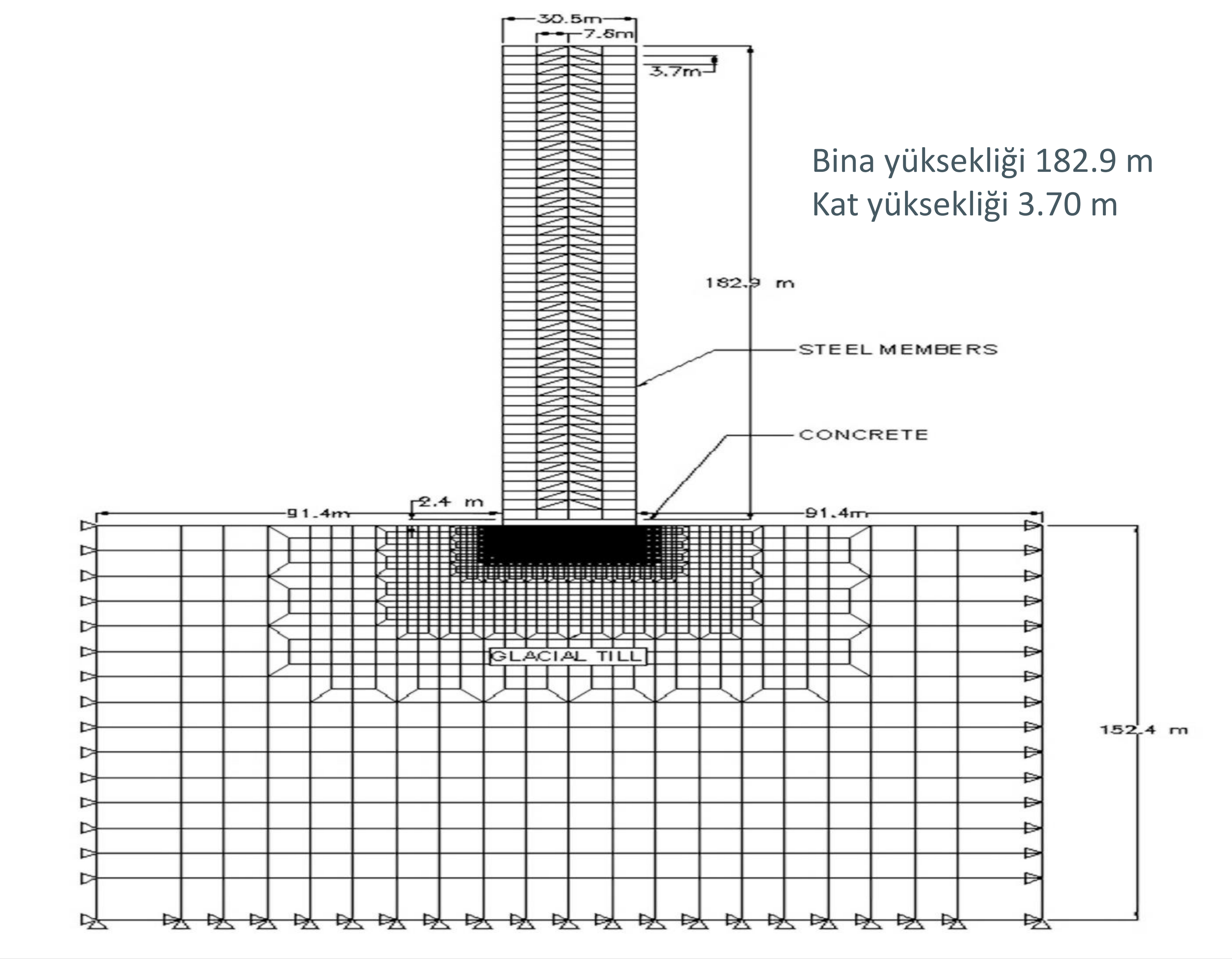
ZEMİNLE İLGİLİ VARSAYIMLAR

* Temel hesabı yapılırken zeminin idealleştirilerek modellenmesi gerekir. Bu tür modellemenin en kapsamlı ve karmaşığı yapı ve zemini birlikte ele alan, yapı-zemin etkileşim modelidir.





Bina yüksekliği 182.9 m
Kat yüksekliği 3.70 m

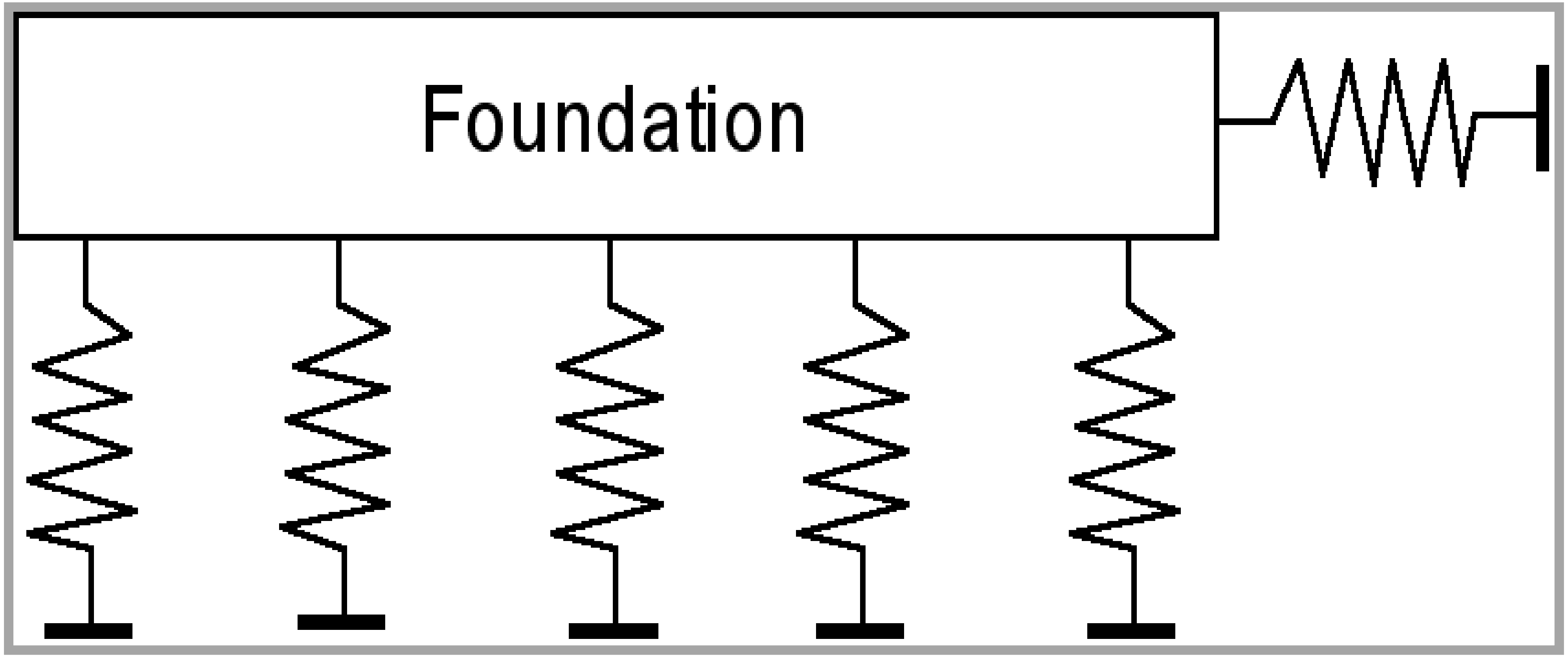


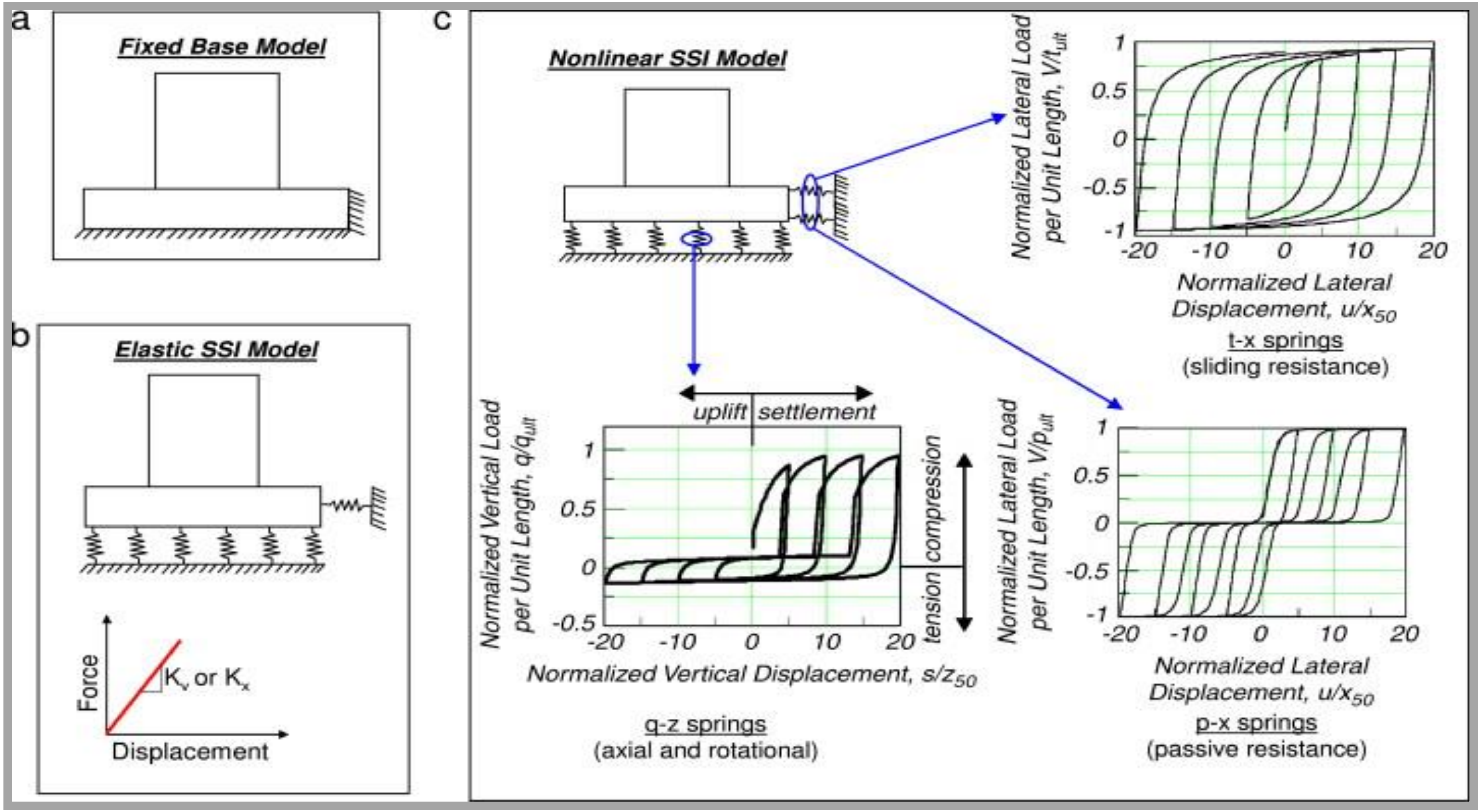
Radye temel ve zemin üzerinde desteklenen elli katlı bir binanın ANSYS modeli

* Diğer bir yöntem, zeminin elastik yaylarla temsil edilmesi, çözümün elastik zemine oturan kiriş ve plak teorisi ile çözümleme yöntemidir.

Bu yöntemde yay sabiti olarak zeminin yatak katsayısı kullanılacağından, önce bu katsayının hesaplanması gerekir. Bu katsayı deneysel verilere dayandırılmalıdır.

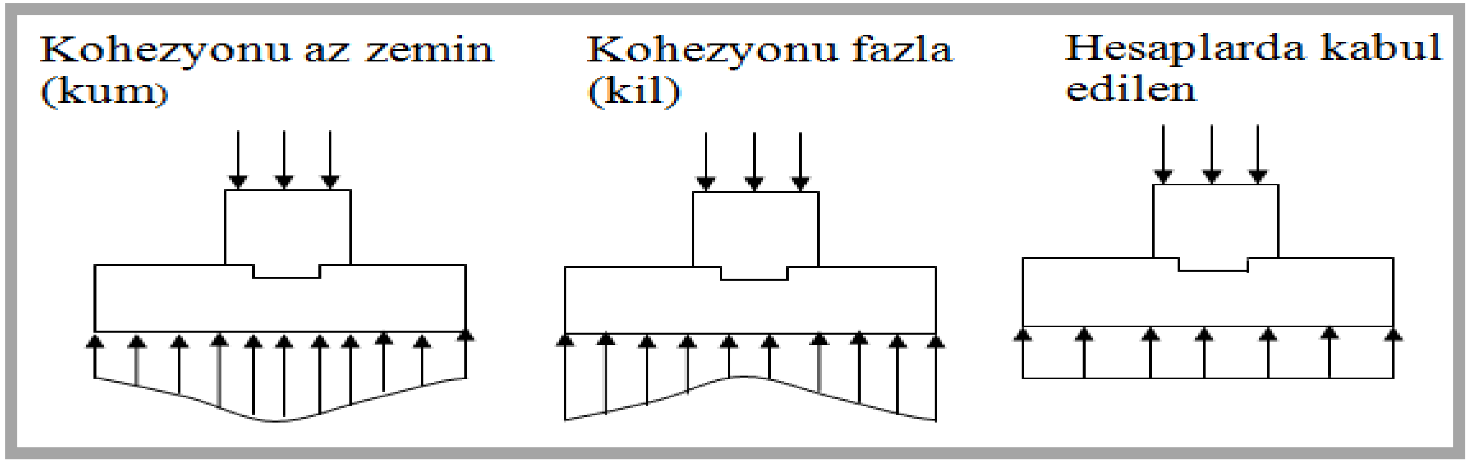
Bu yaklaşım sürekli kolon temelleri ve radyeler için uygulanmaktadır.


Foundation


a
Fixed Base Model
b
Elastic SSI Model
Force
Displacement
K_v or K_x
c
Nonlinear SSI Model
Normalized Lateral Load per Unit Length, V/t_{ult}
1
0.5
0
-0.5
-1
-20
-10
0
10
20
Normalized Lateral Displacement, u/x_{50}
t-x springs
(sliding resistance)
uplift
settlement
Normalized Vertical Load per Unit Length, q/q_{ult}
1
0.5
0
-0.5
-20
-10
0
10
20
tension
compression
Normalized Vertical Displacement, s/z_{50}
q-z springs
(axial and rotational)
Normalized Lateral Load per Unit Length, V/p_{ult}
1
0.5
0
-0.5
-1
-20
-10
0
10
20
Normalized Lateral Displacement, u/x_{50}
p-x springs
(passive resistance)

* Temel hesabında oldukça yaygın olarak kullanılan yöntem, temel altındaki zemin gerilme dağılımı ile ilgili bir varsayım yapmaktır. Bu varsayım oldukça basittir. Genelde gerilme dağılımının düzgün yayılı olduğu, eksantrik yükleme altında ise doğrusal değiştiği kabul edilir. Bu kabul tam doğru değildir.

Tekli kolon temelinin altındaki zemin yaylar ile temsil edildiğinde, oluşacak gerilme dağılımı genelde düzgün yayılı değildir.



ELASTİK MESNETLİ VE BİRLEŞİMLİ SİSTEMLERİN HESABI

$$\delta_{ij} = \int M_i \frac{M_j}{EI} ds + \int N_i \frac{N_j}{EF} ds + \int T_i \frac{T_j}{GF'} ds + \sum M_i \theta_j + \sum P_i v_j$$

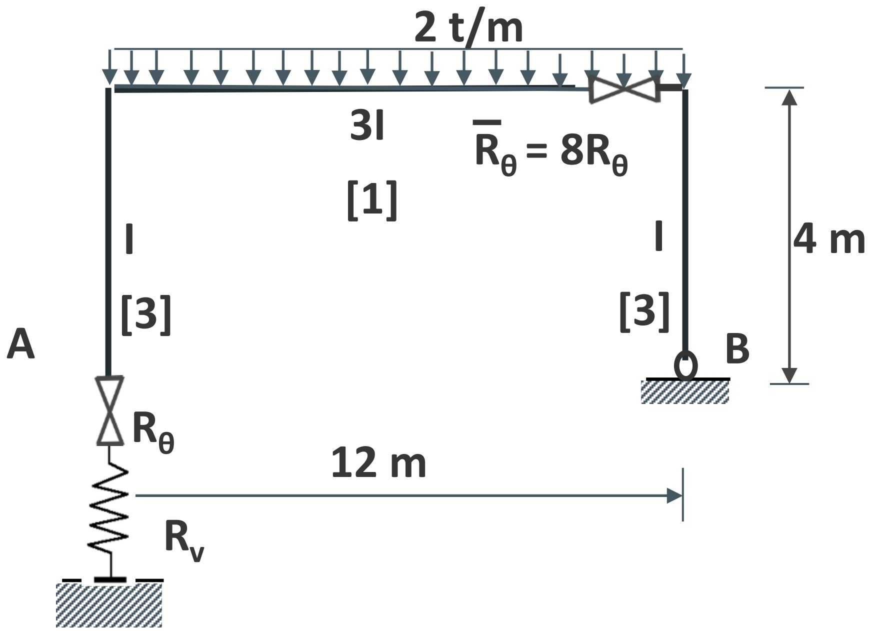
$$\delta_{i0} = \int \mathrm{M_i} \frac{\mathrm{M_0}}{\mathrm{EI}} \mathrm{ds} + \int \mathrm{N_i} \frac{\mathrm{N_0}}{\mathrm{EF}} \mathrm{ds} + \int \mathrm{T_i} \frac{\mathrm{T_0}}{\mathrm{GF'}} \mathrm{ds} + \sum M_i \theta_0 + \sum P_i v_0$$

Kapalı süreklilik denklemlerinde elastik birleşim ve elastik mesnetteki deformasyon işi de göz önünde tutulur.

$$\int M_i \frac{M}{EI} ds + \int N_i \frac{N}{EF} ds + \int T_i \frac{T}{GF'} ds + \int M_i \frac{\varepsilon \Delta t}{d} ds + \int N_i \varepsilon t_s ds + \sum M_i \theta + \sum P_i v = J_i$$

Deformasyon hesaplarında elastik birleşim ve elastik mesnetteki deformasyon işi de göz önünde tutulur.

$$1 \,.\, \delta_t + \sum \bar{R}_0 \Delta = \int \bar{M}_0 \frac{M}{EI} ds + \int \bar{N}_0 \frac{N}{EF} ds + \int \bar{T}_0 \frac{T}{GF'} ds + \int \bar{M}_0 \frac{\varepsilon \Delta t}{d} ds + \int \bar{N}_0 \varepsilon t_s ds + \sum \bar{M}_0 \theta + \sum \bar{P}_0 v$$



Verilenler

Sistem betonarme E=2.1*10^6 t/m^2

I = 80 dm ^{4}

R_v = 19200 t/m R_θ = 12000 tm

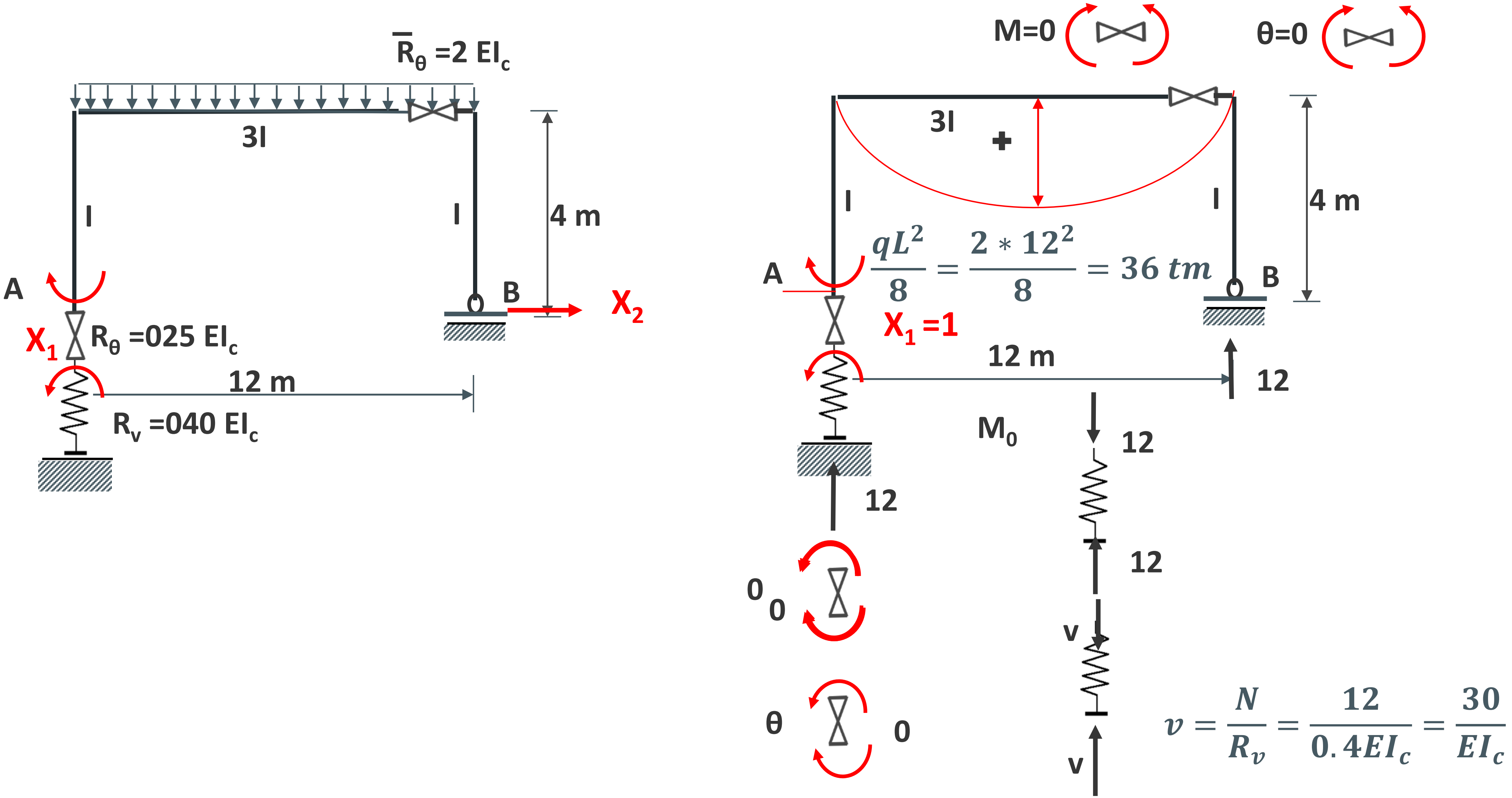
İstenenler

1. **M diyagramı**
2. **Kapalı süreklilik denklemleri**
3. **δ_D = ?**

1. **Moment diyagramı**

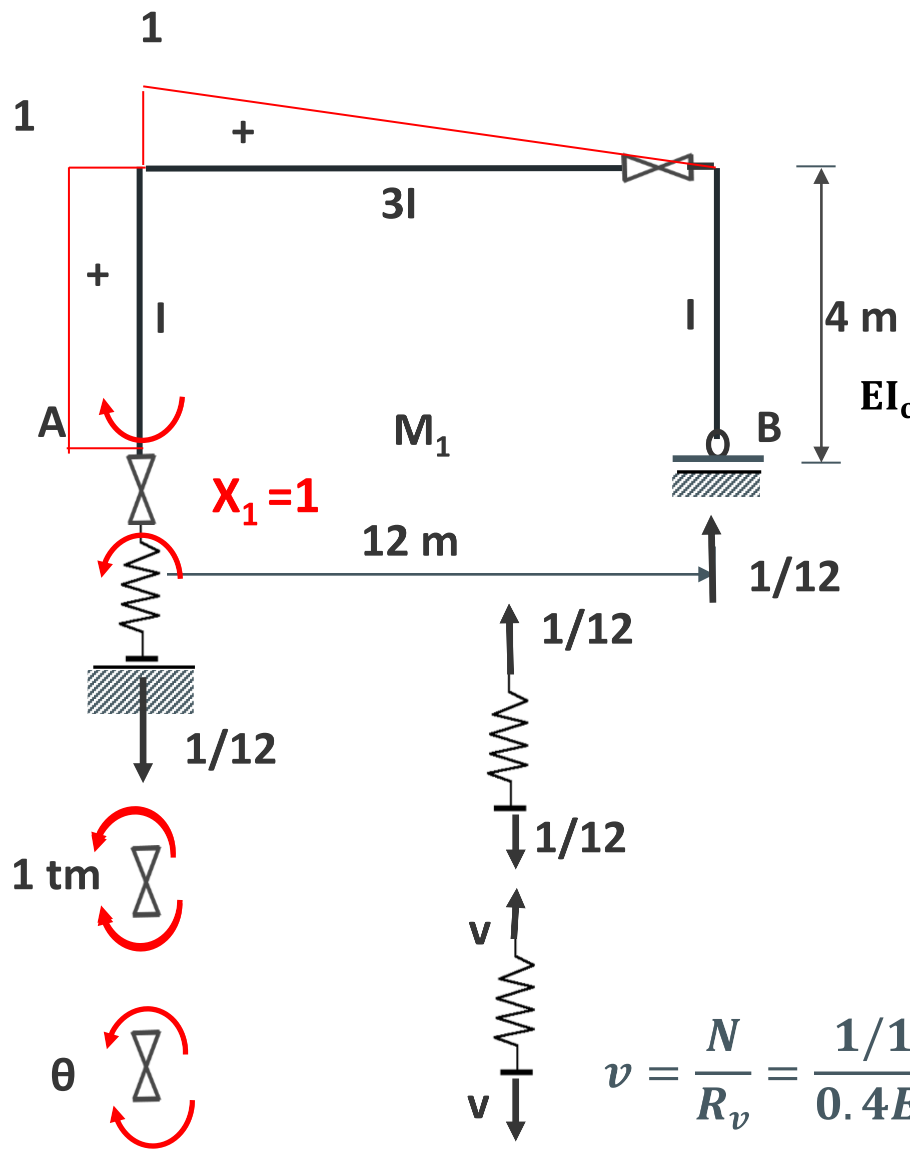
$$I_c = 3I \ \ seçilir \quad EI = 2.1 * 10^6 * 80 * 10^{-4} = 16000 \, tm^2 \quad \frac{R_\theta}{EI} = \frac{12000}{16000} \quad \rightarrow R_\theta = \left(\frac{12000}{16000}\right) EI \quad R_\theta = 0.75EI = 0.25EI_c$$

$$R_v = \left(\frac{19200}{16000}\right) EI = 1.2EI \quad R_v = 0.4EI_c \quad \overline{R}_\theta = 8R_\theta = 6EI = 2EI_c$$



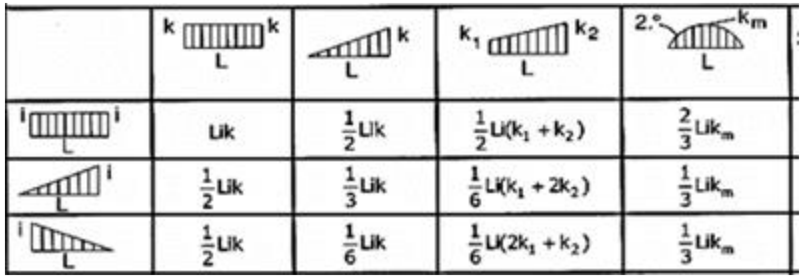
$$\frac{qL^2}{8} = \frac{2 * 12^2}{8} = 36\ tm$$

$$v = \frac{N}{R_v} = \frac{12}{0.4EI_c} = \frac{30}{EI_c}$$



$$EI_c\delta_{11} = 4 * 1 * 1 * [3] + \frac{1}{3} * 12 * [1] * 1 * 1 + EI_c * \overset{M_1 * \theta_1}{\boxed{1 * \frac{4}{EI_c}}} + EI_c * \frac{1}{12} * \overset{P_1 * v_1}{\boxed{\frac{1}{4.8EI_c}}}$$

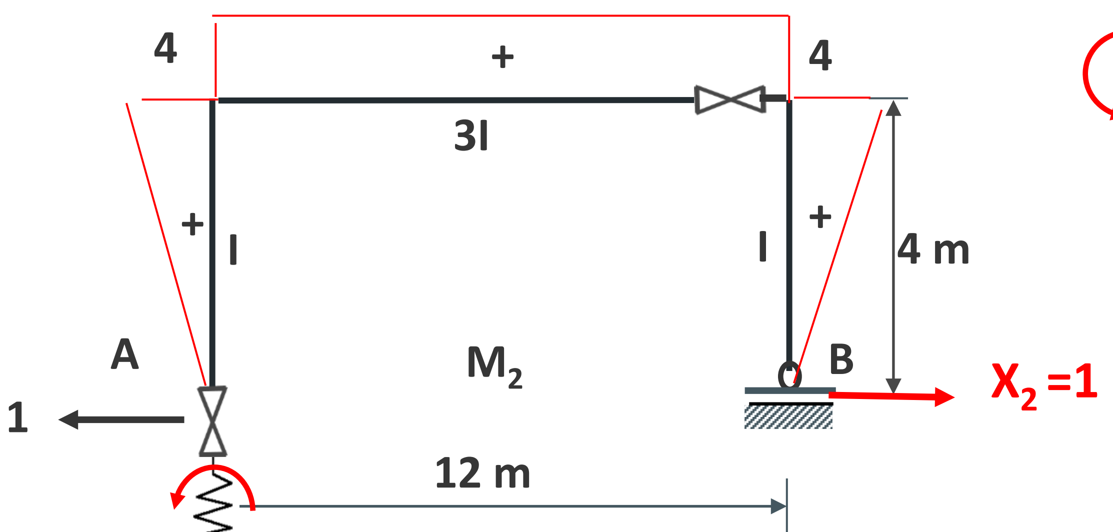
$$EI_c\delta_{11} = 16 + [4 + 0.017] = 20.017$$

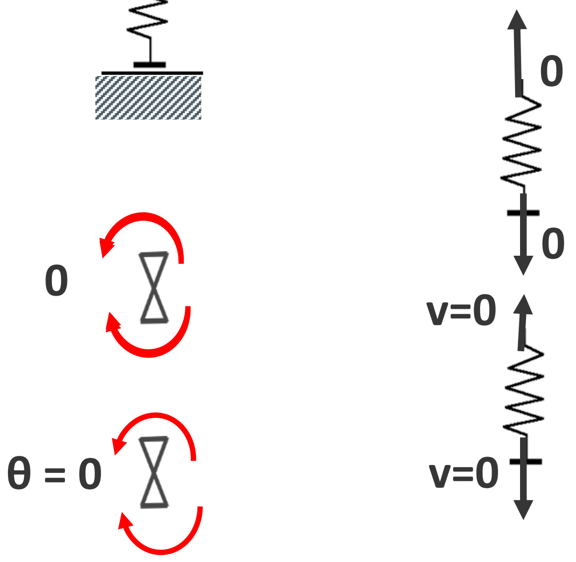


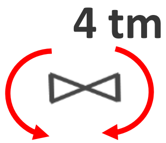
	k ▥ k, L	◿ k, L	k_1 ▥ k_2, L	2.° k_m, L
i ▥ i, L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1 + k_2)	$\frac{2}{3}$Lik$_m$
◿ i, L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1 + 2k_2)	$\frac{1}{3}$Lik$_m$
i ◺, L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k_1 + k_2)	$\frac{1}{3}$Lik$_m$

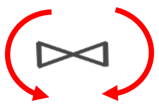
$$v = \frac{N}{R_v} = \frac{1/12}{0.4EI_c} = \boxed{\frac{1}{4.8EI_c}}$$

$$\theta = \frac{M}{R_\theta} = \frac{1}{0.25EI_c} = \boxed{\frac{4}{EI_c}}$$

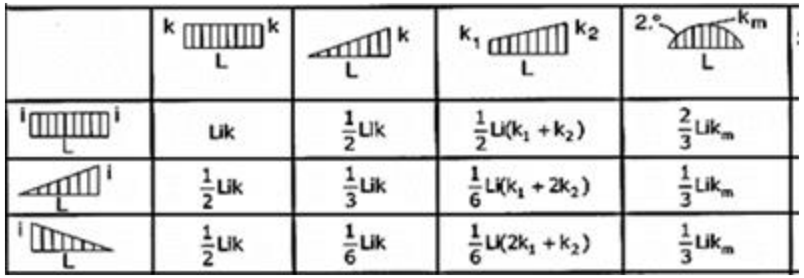








$$\theta = \frac{M}{R_\theta} = \frac{4}{2EI_c} = \frac{2}{EI_c}$$



	k ▭ k, L	◺ k, L	k₁ ▭ k₂, L	2.° ⌒ k_m, L
i ▭ i, L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k₁ + k₂)	$\frac{2}{3}$Lik_m
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i ◸, L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li(2k₁ + k₂)	$\frac{1}{3}$Lik_m

$M_1 * \theta_2$ $P_1 * v_2$ $M_1 * \theta_2$

$$\mathrm{EI_c}\delta_{12} = \frac{1}{2}4 * 1 * 4 * [3] + \frac{1}{2} * 12 * 1 * 4.[1] + \mathrm{EI_c}[1 * 0 + \frac{1}{12} * 0 + 0 * \frac{2}{\mathrm{EI_c}}] = 48$$

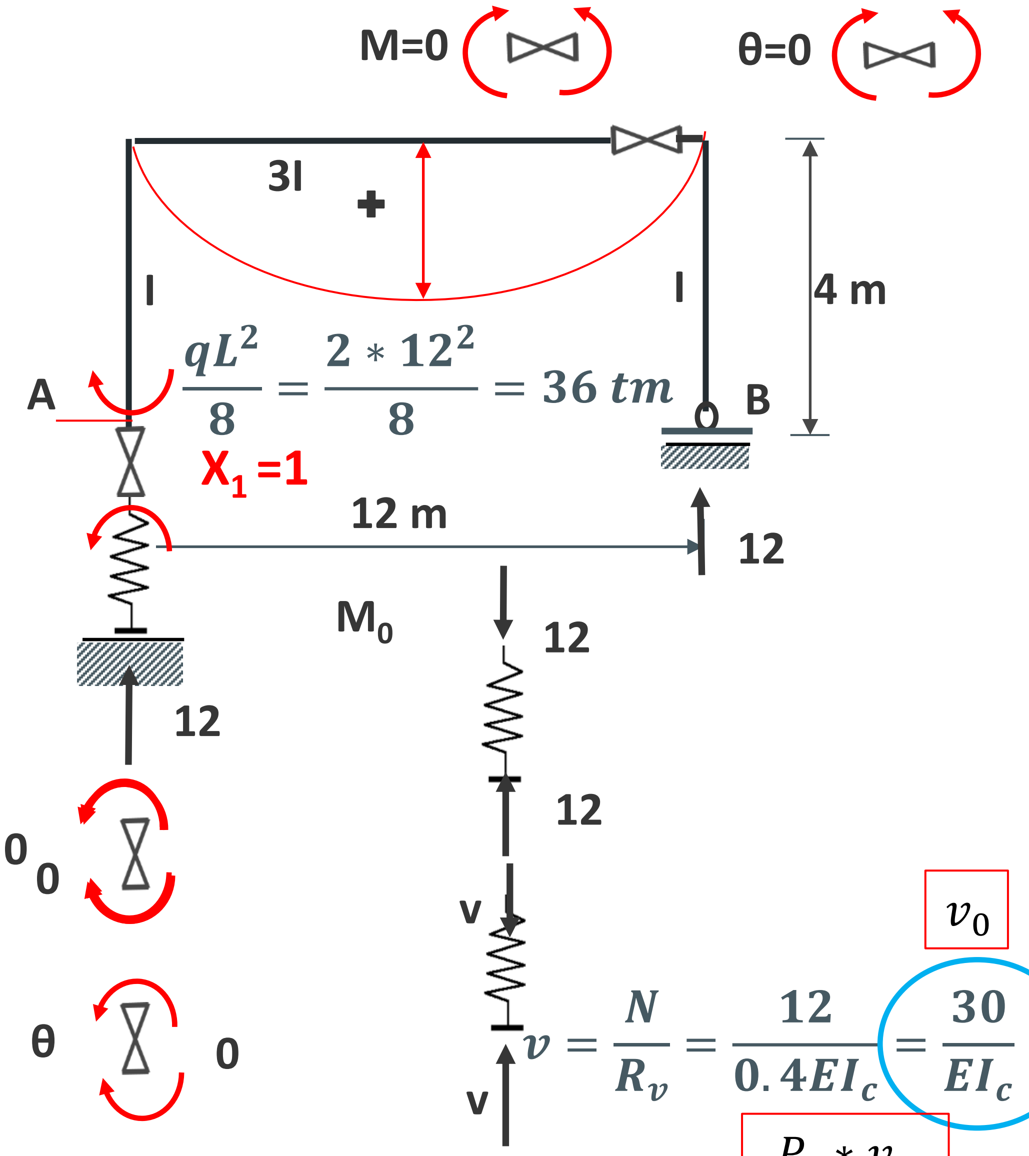
$$\mathrm{EI_c}\delta_{21} = \mathrm{EI_c}\delta_{12} = 48$$

$M_2 * \theta_2$ $P_2 * v_2$ $M_2 * \theta_2$

$$\mathrm{EI_c}\delta_{22} = 2\frac{1}{3}4 * 4 * 4 * [3] + 12 * 4 * 4 * [1] + \mathrm{EI_c}[0 + 0 + 4 * \frac{2}{\mathrm{EI_c}}] = 328$$

$$\mathrm{EI_c}\delta_{10} = \frac{1}{3}12 * 1 * 36 * [1] + \mathrm{EI_c}\left[-\frac{1}{12} * \frac{30}{\mathrm{EI_c}}\right] = 141.5$$

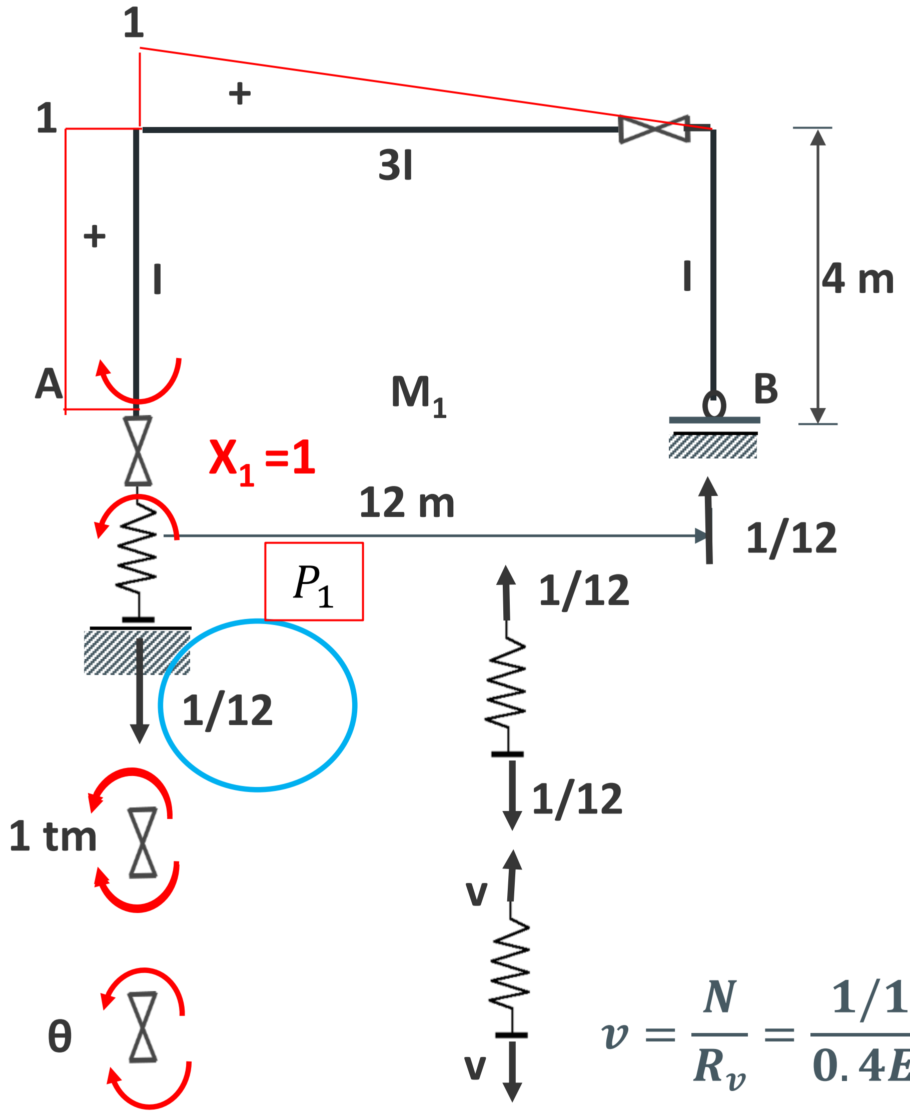
$$\mathrm{EI_c}\delta_{20} = \frac{2}{3}12 * 36 * 4 * [1] + \mathrm{EI_c}[0 * 4] = 1152$$



$$\frac{qL^2}{8} = \frac{2 * 12^2}{8} = 36\ tm$$

$$v = \frac{N}{R_v} = \frac{12}{0.4EI_c} = \boxed{\frac{30}{EI_c}}$$

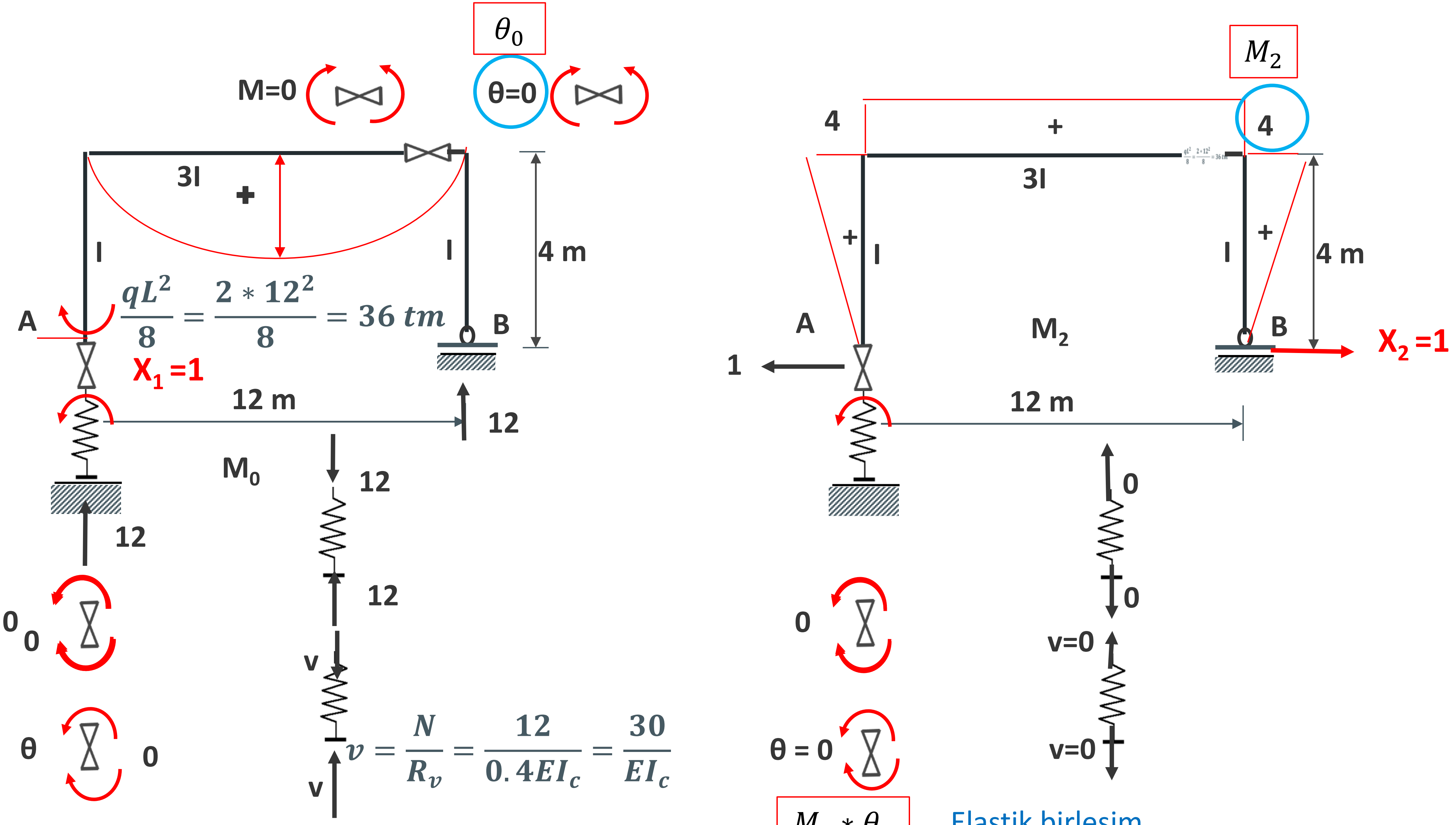
$\boxed{v_0}$



$$v = \frac{N}{R_v} = \frac{1/12}{0.4EI_c} = \frac{1}{4.8EI_c}$$

$$\theta = \frac{M}{R_\theta} = \frac{1}{0.25EI_c} = \frac{4}{EI_c}$$

$$\mathrm{EI_c}\delta_{10} = \frac{1}{3}12 * \ 1 \ * 36 * \ [1] + \mathrm{EI_c}\overset{\boxed{P_1 * v_0}}{\left[-\frac{1}{12} * \frac{30}{\mathrm{EI_c}}\right]} = 141.5$$



$$\frac{qL^2}{8} = \frac{2*12^2}{8} = 36\ tm$$

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$$\boxed{M_2 * \theta_0}$$

Elastik birleşim

$$\mathbf{EI_c\delta_{20} = \frac{2}{3}12*36*\ 4*[1] + EI_c[4*0] = 1152}$$

Açık süreklilik denklemleri

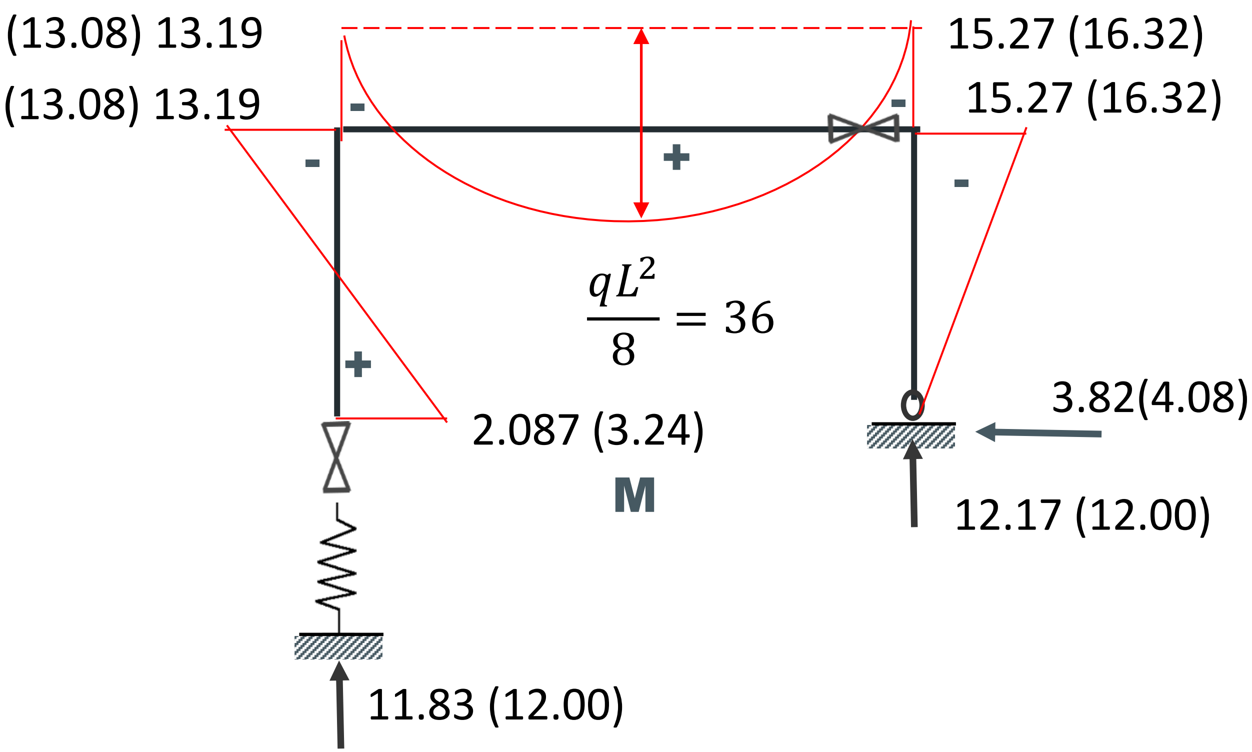
$$EI_c\begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}\begin{Bmatrix}X_1\\ X_2\end{Bmatrix}+EI_c\begin{Bmatrix}\delta_{10}\\ \delta_{20}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$\mathbf{20.047X_1+48X_2+141.5=0}$$

$$\mathbf{48X_1+328X_2+1152=0}$$

$$\mathbf{X_1=2.087\ tm \qquad X_2=-3.818\ t}$$

Parantez içindeki değerler sistemin elastik mesnetsiz aynı yükler altında çözümüdür.



2. Kapalı süreklilik denklemleri

1 nolu kapalı süreklilik denklemi

$$\int M_1 M \frac{I_c}{I} ds + EI_c \left[\sum M_1 \theta + \sum P_1 v \right] = 0$$

$$\frac{1}{2} * 4 * 1 * (2.087 - 13.19)[3] + \frac{1}{6} * 12 * 1 * [2 * (-13.19) - 15.27][1] + \frac{1}{3} 12 * 1 * 36 * [1]$$

$$+ EI_c \left[1 * \frac{2.087}{0.25EI_c} - \frac{1}{12} \frac{11.83}{0.40EI_c} \right] = 0\ ?$$

$$-66.6 - 83.3 + 144.0 + 8.36 - 2.46 = -152.35 + 152.36 = 0 \quad \surd$$

2 nolu kapalı süreklilik denklemi

$$\int M_2 M \frac{I_c}{I} ds + EI_c \left[\sum M_2 \theta + \sum P_2 v \right] = 0$$

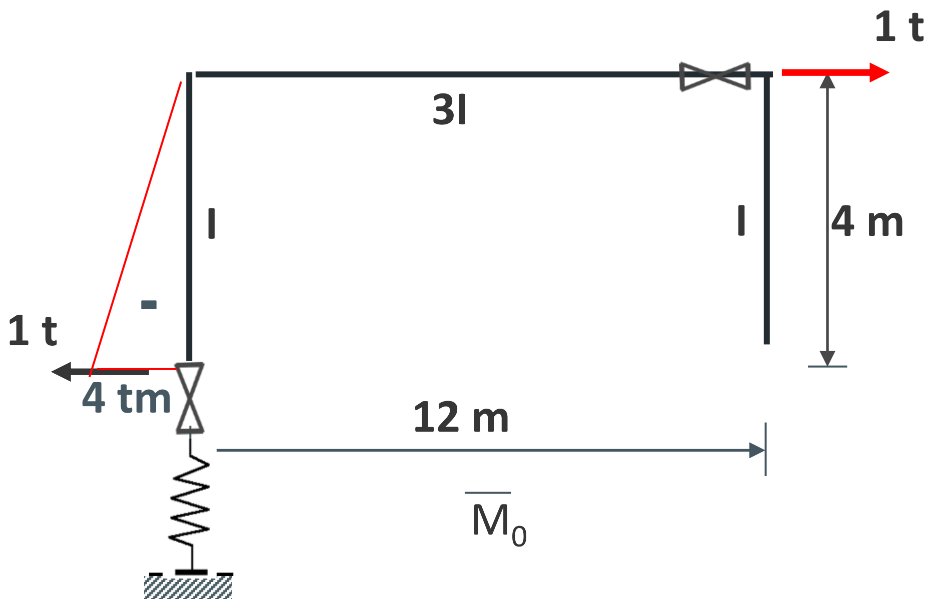
$$\frac{1}{6} * 4 * 4 * (2.087 - 2 * 13.19)[3] + \frac{1}{2} * 12 * 4 * [(-13.19) - 15.27][1] + \frac{2}{3} 12 * 4 * 36[1] + \frac{1}{3} * 4 * 4$$

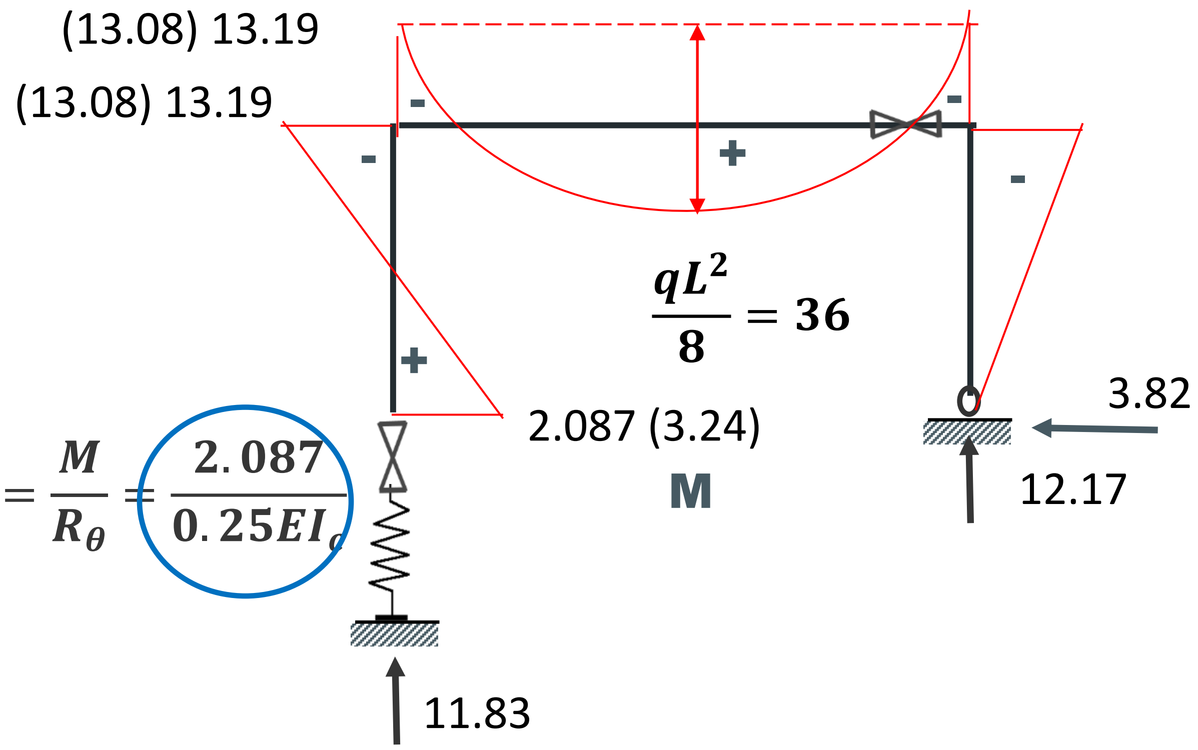
$$* (-15.27)[EI_c + EI_c \left[4 * \frac{-15.27}{8EI_c} \right] = 0\ ?$$

$$-104.32 - 683.04 + 1152 - 244.32 - 30.54 = -1152.21 + 1152 = -0.22 \qquad Relatif\ hata$$

$$= \frac{0.22}{1152} = \%0.13 \quad \surd$$

3. $\delta_D = ?$ D noktasının yatay deplasmanını bulunuz





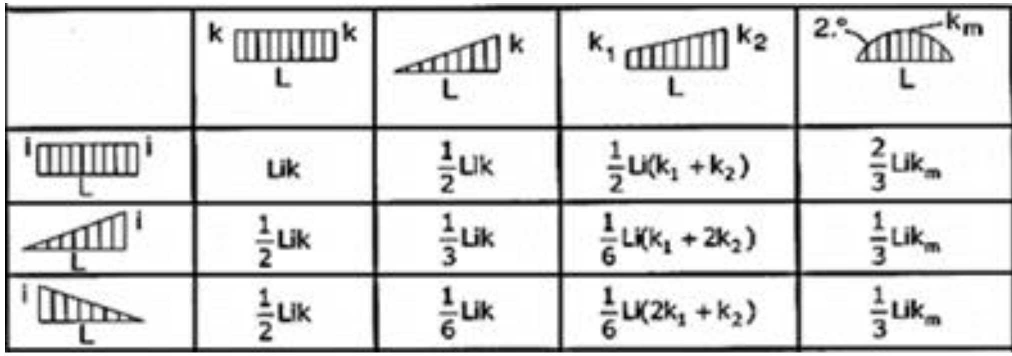
$$\theta = \frac{M}{R_\theta} = \frac{2.087}{0.25EI_c}$$

$$1 * \delta_D = \int \overline{M}_0 \frac{M}{EI} ds + \sum \overline{M}_0 \theta$$

$$EI_c\delta = \int M\,\overline{M}_0 \frac{I_c}{I} ds + EI_c \sum \overline{M}_0 \theta$$

$$EI_c\delta_D = \frac{1}{6} 4 * (-4)(2 * 2.087 - 13.19)\,[3] + EI_c\left[(-4)\frac{2.087}{0.25EI_c}\right] = 38.736$$

$$\rightarrow \delta_D = \frac{38.736}{EI_c} = \frac{38.736}{E3I} = \frac{12.912}{EI}$$



	k k L	k L	k_1 k_2 L	2.° k_m L
i i L	Lik	$\frac{1}{2}$Lik	$\frac{1}{2}$Li(k_1+k_2)	$\frac{2}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{3}$Lik	$\frac{1}{6}$Li(k_1+2k_2)	$\frac{1}{3}$Lik$_m$
i L	$\frac{1}{2}$Lik	$\frac{1}{6}$Lik	$\frac{1}{6}$Li($2k_1+k_2$)	$\frac{1}{3}$Lik$_m$